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A Computer-assisted existence proof for Emden's equation on an unbounded domain via eigenvalue bounds

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We prove existence, non-degeneracy, and exponential decay at infinity of a non-trivial solution to Emden's equation $-\Delta u = |u|^3$ on an unbounded L-shaped domain, subject to Dirichlet boundary conditions. Our proof is computer-assisted: Starting from a numerical approximate solution, we use a fixed-point argument to prove existence of a near-by exact solution. The most crucial step in this argument is the computation of a norm bound for the inverse of the linearization at the approximate soution. We propose to use eigenvalue bounds for this purpose, which in turn we compute by variational methods supported by a homotopy approach. These eigenvalue bounds also imply non-degeneracy of the solution.

This is joint work with Filomena Pacella, and Dagmar Rütters.

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