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Symmetries in Port-based Teleportation Schemes

Michał Studzińsk *

studzinski.m.g@gmail.com

The port-based teleportation (PBT) protocol introduced in 2008 by Ishizaka and Hiroshima is a variant of quantum teleportation scheme which transmits the unknown state to the receiver *without requiring any corrections* on his/her side [**?**]–in the opposite to the original teleportation presented in [**?**]. In the primary setting in this protocol, the sender and the receiver share N copies of the maximally entangled states (*resource state*), where each singlet is a two-qubit state, called port. The sender implements a joint POVM on the teleported system and the resource state.

The lack of mentioned correction allows for many important applications such as engineering efficient protocols for instantaneous implementation of measurements and computation [**?**], communication complexity [**?**], some implications on limitations on quantum channels discriminations, and quantum messages compression [**?**].

Unfortunately, evaluating the performance of PBT was computationally intractable, and up to now all attempts succeeded only with small systems - qubit case ($d = 2$) or small number of ports N. In the qubit case, solution relies on the existence of the closed form for the Clebsch-Gordan coefficients, and therefore was limited only to $SU(2)^{\otimes N}$ [**?**]. In the latter case, relies on a correspondence with a graphical variant of the Temperley-Lieb algebra which worked for an arbitrary d but required exponential computational overhead in the number of ports with explicit expressions available for $N \leq 5$ [**?**].

An open problem was to find optimal PBT protocols with establishing fundamental limits on its performance in the most general setting, i.e. the optimal measurement that works for an arbitrary dimension and number of ports and parties are not restricted to N copies of maximally entangled states.

In our work we *fully characterize* the performance of *all* existing PBT protocols in arbitrary local dimensions d as well as arbitrary number of ports N. I turns out that operators describing the performance of PBT belong to recently-studied by the authors algebra of

*Institute of Theoretical Physics and Astrophysics, National Quantum Information Centre, Faculty of Mathematics, Physics and Informatics, University of Gdańsk, 80-308 Gdańsk, POLAND

partially transposed permutation operators [**?**]. Such property allows us to use tools coming form the representation theory of finite groups and algebras, and analyze the problem on every irreducible space separately. We are able to find optimal probabilistic and deterministic versions of the PBT protocols in the most general case. In the latter case, the answer depends only on the largest eigenvalue of a particular matrix, so called *teleportation matrix*, which encodes the relationship between a set of Young diagrams and emerges as the optimal solution of the semidefinite program. Additionally, we solve an eigenproblem for generators of algebra which is the first step towards to hybrid port-based teleportation scheme and gives us new proofs of asymptotic behaviour of teleportation fidelity.

Summarizing, our results rely on two key innovations: 1) a novel connection between the operators from PBT and the recently-studied algebra of partially transposed permutation operators, and 2) a theory of partially reduced irreducible representations (PRIR) – a new tool to efficiently compute products of operators which possess partial symmetries.

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This is joint work with Marek Mozrzymas (Institute for Theoretical Physics, University of Wrocław), Sergii Strelchuk (DAMTP, Centre for Mathematical Sciences, University of Cambridge) and Michał Horodecki (Institute of Theoretical Physics and Astrophysics, National Quantum Information Centre, University of Gdańsk).