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Five-point differential-difference equations: Their growth properties and stationary reductions

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In this talk we consider Volterra-like five-point differential-difference equations, that is differential-difference equations of the following form:

$$\dot{u}_n = A\left(u_{n+1}, u_n, u_{n-1}\right) u_{n+2} + B\left(u_{n+1}, u_n, u_{n-1}\right) u_{n-2} + C\left(u_{n+1}, u_n, u_{n-1}\right), \qquad n \in \mathbb{Z},\ t \in \mathbb{C}. \tag{1}$$

Integrable equations of the form (1) were recently classified in [3, 4] using the generalised symmetry method [7] as integrability criterion.

We show that, when applicable, the equations found in [3, 4] are integrable also according to the algebraic entropy criterion [1, 6], that is their algebraic entropy is zero and their rate of growth is polynomial.

We then consider the stationary reductions of the equations found in [3, 4], i.e. we consider the case when $\dot{u}_n \equiv 0$, which reduces a differential-difference equation (1) to a fourth-order difference equation. We study the integrability properties of these stationary reductions in the sense of algebraic entropy and in the sense of the existence of *invariants*. We comment on the relationship with known classes of integrable fourth-order difference equations [2, 5].

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