\( \mathbb{C}P^{2S} \) sigma models described through hypergeometric orthogonal polynomials

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The main objective of this talk is to establish a new connection between the Hermitian rank-1 projector solutions of the Euclidean \( \mathbb{C}P^{2S} \) sigma model in two dimensions and the particular hypergeometric orthogonal polynomials called Krawtchouk polynomials. We show that any such projector solutions of the \( \mathbb{C}P^{2S} \) model, defined on the Riemann sphere and having a finite action, can be explicitly parametrised in terms of these polynomials. We apply these results to the analysis of surfaces associated with \( \mathbb{C}P^{2S} \) models defined using the generalised Weierstrass formula for immersion. We show that these surfaces are homeomorphic to spheres in the \( \mathfrak{su}(2s+1) \) algebra, and express several other geometrical characteristics in terms of the Krawtchouk polynomials. Finally, a connection between the \( \mathfrak{su}(2) \) spin-s representation and the \( \mathbb{C}P^{2S} \) model is explored in detail. It is shown (Proposition 5.2) that for any given holomorphic vector function in \( \mathbb{C}^N \), it is possible to derive solutions of the \( \mathbb{C}P^{2S} \) model through algebraic recurrence relations which turn out to be simpler than the analytic relations known from the literature.