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"Integrability: Continuous and discrete, classical & quantum"

On series of infinite families of superintegrable systems separating in polar coordinates

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In classical mechanics, a Hamiltonian system with n degrees of freedom is called integrable (or Liouville integrable) if, in addition to the Hamiltonian, it allows $(n-1)$ Poisson commuting well defined integrals of motion. This system is said to be superintegrable if it admits further integrals. In the quantum case, to define the notion of integrability the integrals (functions) on phase space must be replaced by Hermitian operators on a Hilbert space, and the Poisson bracket by the commutator Lie bracket.

We will present a general description of superintegrable systems, in a 2-dimensional Euclidean space, that have the following properties: 1. They allow separation of variables in polar coordinates and 2. They admit an independent polynomial integral of motion Y of order $N \geq 2$.

The corresponding potentials can be classified into two major classes: the standard potentials which satisfy nontrivially a linear compatibilty condition (a linear ODE) for the existence of the integral Y, and the exotic ones for which such equation is trivially fulfilled. It is conjectured that all the exotic potentials obey a nonlinear ODE that has the Painlevé property. The main characteristics of both classes of potentials are described. In particular, we will see that the well-known Tremblay–Turbiner–Winternitz (TTW) model fits nicely in the class of standard potentials. In quantum mechanics, we present an infinite family of superintegrable potentials in terms of the sixth Painlevé transcendent P_6. We will discuss the classical analogs of these potentials as well.

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