

# Darboux-Bäcklund transformations for Spin-valued linear problems

Jan Cieśliński\*

[j.cieslinski@uwb.edu.pl](mailto:j.cieslinski@uwb.edu.pl)

---

We consider integrable systems associated with linear problems taking values in Spin groups, see [1]. Our main motivation to study this case came from differential geometry of submanifolds [2]. It turned out that Clifford algebras and Spin groups are convenient tools to simplify the problem of constructing the Darboux-Bäcklund transformation in the case of the so called isothermic surfaces, where the group  $SO(4,1)$  is involved, compare [3] and [4]. Note that geometric problems in the 3-dimensional space are usually related to the group  $SO(3)$  and even in that case one always uses in fact the corresponding Spin group, i.e.  $SU(2)$  which is isomorphic to  $Spin(3)$ .

We focus on reduction groups which can be represented by standard automorphisms and anti-automorphisms in Clifford algebras (grade involution, reversion and Clifford conjugation). Determinants, crucial in the matrix approach, are replaced by the spinor norm. It is remarkable that the presented algebraic theory simplifies greatly the derivation of the Darboux transformation in the case of Darboux matrices linear in the spectral parameter. The general case is much more difficult. We present new results and some open problems.

- [1] J. L. Cieśliński, A class of linear spectral problems in Clifford algebras, *Physics Letters A* 267 (2000) 251–255.
- [2] J. Cieśliński, The Darboux-Bianchi transformation for isothermic surfaces. Classical results versus the soliton approach, *Differential Geometry and Its Applications* 7 (1997) 1–28.
- [3] W. Biernacki, J. L. Cieśliński, A compact form of the Darboux-Bäcklund transformation for some spectral problems in Clifford algebras, *Physics Letters A* 288 (2001) 167–172.
- [4] J. L. Cieśliński, Geometry of submanifolds derived from Spin-valued spectral problems, *Theoretical and Mathematical Physics* 137 (2003) 1396–1405.

---

\*Faculty of Physics, University of Białystok, ul. Ciołkowskiego 1L, Białystok 15-196, POLAND