

Color algebraic extension of supersymmetric quantum mechanics

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In a recent paper [1], Bruce and Duplij introduced a double-graded version of supersymmetric quantum mechanics (SQM). It is a model of quantum mechanical Hamiltonian H which can be factorized in two distinct way, i.e., $H = Q_{01}^2 = Q_{10}^2$. However, this is not a $\mathcal{N} = 2$ SQM since the two supercharges Q_{01}, Q_{10} close in the commutator to give a central element of the algebra. As a result, the algebra generated by H and supercharges is not a Lie superalgebra, but a color superalgebra of $\mathbb{Z}_2^2 = \mathbb{Z}_2 \times \mathbb{Z}_2$ grading introduced about 40 years ago [2, 3].

The Bruce-Duplij model may be regarded as a color superalgebraic extension of $\mathcal{N} = 1$ SQM. In the present work, we present an extension of the Bruce-Duplij model to higher values of \mathcal{N} . More precisely, it is shown that one may obtained a double-graded SQM starting from the ordinary SQM of Akulov and Kudinov with any even value of \mathcal{N} . Furthermore, it is shown that the present extension of SQM to a double-graded one is a special case of a construction of \mathbb{Z}_2^2 -graded color superalgebra based on an ordinary Lie superalgebra. Therefore, one may extend any superconformal mechanics (if it satisfy some conditions) to its double-graded version. We discuss a typical example in some detail.

- [1] A. J. Bruce and S. Duplij, *Double-graded supersymmetric quantum mechanics*, arXiv:1904.0697.
- [2] V. Rittenberg and D. Wyler, *Generalized Superalgebras*, Nucl. Phys. B **139**, 189 (1978) .
- [3] V. Rittenberg and D. Wyler, *Sequences of $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ graded Lie algebras and superalgebras*, J. Math. Phys. **19**, 2193 (1978).

This is joint work with K. Amakawa and S. Doi.

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