Programme du 50e anniversaire du CRM
« Atelier sur les nouvelles tendances en systèmes différentiels polynomiaux »
3 au 7 septembre 2019

CRM 50th anniversary program
"Workshop on New Trends in Polynomial Differential Systems"
September 3-7, 2019

Global geometrical classification of singularities

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During the last eight years, J. Llibre, D. Schlomiuk, N Vulpe and myself have been working on a complete classification of singularities of planar quadratic differential systems. A few previous studies had already been done by us when we began. We wanted to go deeper than the usual topological classification in order to take care of algebraic properties important for studying bifurcations, such as degree of weak foci or saddles, multiplicity of singularities or tangential behavior of orbits.

We needed to define a new concept: the geometrical equivalence relation of singularities and the resulting geometrical classification of the whole set of configurations of singularities (finite and infinite), including all degenerate singularities and even systems with an infinite number of singularities. We also introduced a notation for singularities such that it describes by means of a simple code, all of the geometrical properties of singularities of quadratic differential equations.

Our study has produced 1765 different geometric configurations of singularities which can be distinguished in terms of invariant polynomials. The complete bifurcation diagram in the 12-dimensional space $\mathbb{R}^12$ of parameters of the global geometrical configurations of singularities, is obtained in terms of invariant polynomials.

A Mathematica program to compute the invariant polynomials and the configuration of singularities for any system presented in any normal form, is also provided. This forms an algorithm that can be applied to families with a few parameters.

From this geometrical classification it is easy to extract a topological classification of configurations of singularities. We end up with 208 topological configurations.

And finally these 208 topological configurations form a nice skeleton over which the set of different topological phase portraits may start to be obtained. Many of these configurations lead to a single phase portrait, but there are already completed studies for specific

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configurations which give up to 97 phase portraits (modulo limit cycles). Over 100 of these 208 topological configurations have already been studied providing about 800 distinct phase portraits modulo limit cycles.