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Geometric and topological recursion

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Initially inspired by the separate works of Mirzakhani and Eynard, the geometric recursion is a recipe proposed last year together with Andersen and Orantin, which aims at constructing mapping class group invariants for surfaces of arbitrary topologies, from a small amount of initial data, by induction on the Euler characteristic. It can be used, for instance, to construct certain functions on the moduli space of bordered Riemann surfaces, which are such that their integration over the moduli space gives functions of boundary lengths that satisfy a variant of the topological recursion of Chekhov, Eynard and Orantin. One can produce in this way functions which are relevant in enumerative geometry and hyperbolic geometry, such as the constant function 1 (which yield, after integration, the Weil-Petersson volumes of the moduli space) and linear statistics of the hyperbolic length of simple closed curves. Besides, any initial data for the topological recursion can be lifted to an initial data for the geometric recursion, so that the topological recursion amplitudes are recovered by integration of the corresponding geometric recursion amplitude. This gives a refined and an hyperbolic perspective on certain classical problems of enumerative geometry. The meaning of this refined information and the perspectives opened by this construction are still to be explored.

In this lecture, I will give an introduction to the geometric recursion and its main properties. As illustration, I will revisit some problems solved by the topological recursion in its light, explain new applications in the context of hyperbolic geometry, and indicate natural questions in geometry and topological field theories that one can think of addressing via his formalism.

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