Programme du 50e anniversaire du CRM
« Atelier sur les facettes de l'intégrabilité »
6 au 10 mai 2019

CRM 50th anniversary program
"Workshop on Faces of Integrability"
May 6-10, 2019

Tilings, paths and arctic curves: The tangent method at work

Philippe R. Di Francesco *

philippe@illinois.edu

Tiling problems of finite domains of the plane with a fixed set of tiles can often be rephrased in terms of non-intersecting lattice paths. For large scaled domains, random tilings can exhibit a sharp separation between "frozen" regions tiled regularly and "liquid" regions tiled wildly. This is the arctic phenomenon. The separating curve is called "arctic curve".

We present a new technique due to Colomo and Sportiello, called the tangent method, to derive the arctic curve using only boundary properties of the set of paths describing the tilings. We apply this technique to domino tiling problems of Aztec rectangles with prescribed defects. We perform exact enumerations using the Gessel-Viennot determinant for non-intersecting lattice paths, and asymptotic analysis. This leads to compact expressions for arctic curves and their q-deformations in the presence of area-dependent weights. Time permitting, we will also discuss generalizations to osculating paths.

Based on joint works with M.F. Lapa and E. Guitter.

*Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street, (MC-382), Urbana, IL 61801, USA