Symplectic extensions of the Kirillov-Kostant and Goldman Poisson structures and Fuchsian systems

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We revisit symplectic properties of the monodromy map for Fuchsian systems on the Riemann sphere. We extend previous results in the literature where it was shown that the monodromy map is a Poisson morphism between the Kirillov-Kostant Poisson structure on the space of coefficients, on one side, and the Goldman bracket on the monodromy character variety on the other. The extension is provided by defining larger spaces on both sides which are equipped with symplectic structures naturally projecting to the canonical ones. On the coefficient side our symplectic structure corresponds to a non-degenerate quadratic Poisson structure expressed via the rational dynamical $r$-matrix; it reduces to the Kirillov-Kostant bracket when projected to the standard space. On the monodromy side we get a symplectic structure which induces the same symplectic structure induced by the Goldman bracket on its symplectic leaves. As a corollary we prove the recent conjecture by A. Its, O. Lisovyy and A. Prokhorov in its "strong" version while the original "weak" version is derived from previously known results. We show also that the isomonodromic Jimbo-Miwa tau-function is intimately related to a generating function of such transformation.

Joint work (in progress) with Dmitry Korotkin.