Programme du 50^{e} anniversaire du CRM
« Atelier sur les facettes de l'intégrabilité »
6 au 10 mai 2019

CRM 50th anniversary program
"Workshop on Faces of Integrability"
May 6-10, 2019

KP theory, plabic networks in the disk and rational degenerations of M–curves

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In this talk I shall present some recent results (see arXiv:1801.00208v2, arXiv:1805.05641) obtained in collaboration with P.G. Grinevich (LITP, RAS) on the connection between totally non-negative Grassmannians and the reality problem in KP finite-gap theory via the assignment of real regular divisors on rational degenerations of M-curves for the class of real regular multi-line soliton solutions of Kadomtsev-Petviashvili II (KP) equation whose asymptotic behavior in space-time has been combinatorially characterized in a series of papers by S. Chakravarthy, Y. Kodama and L. Williams. At this aim, we use the planar bicolored networks in the disk which were introduced by A. Postnikov to parametrize positroid cells in totally nonnegative Grassmannians. In our construction the boundary of the disk corresponds to the rational curve associated to the soliton data in the direct spectral problem, and the bicolored graph is the dual of a reducible curve G which is the rational degeneration of a regular M-curve whose genus g equals the number of faces of the network diminished by one. We then assign systems of edge vectors to the planar bicolored networks. The system of relations satisfied by these vectors has maximal rank and may be reformulated in the form of edge signatures as proposed by T. Lam. Adapting remarkable results by A. Postnikov and K. Talaska to our setting, we prove that the components of the edge vectors are rational in the edge weights with subtraction free denominators and provide their explicit expressions in terms of conservative and edge flows. The edge vectors rule the value of the KP wave function at the double points of G, whereas the signatures at the vertices rule the position of the divisor points in the ovals. In particular, we provide a combinatorial proof that the degree g divisor satisfies the conditions settled by B. Dubrovin and S. Natanzon for real finite-gap solutions, i.e. there is exactly one divisor point in each finite oval and no divisor point in the oval containing the essential singularity of the wave function. The divisor points may be explicitly computed using the linear relations satisfied by the wave function at the internal vertices of the chosen network. Finally we explain the role of moves and reductions in the transformation of both the curve and the divisor for

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given soliton data, and we apply our construction to some examples.