Programme du 50^e
Atelier sur les variétés de carquois et la théorie des représentations
12 au 16 août

50th anniversary program
Workshop on quiver varieties and representation theory
August 12-16, 2019

Tensor Product Multiplicity via Upper Cluster Algebras

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By *tensor product multiplicity* we mean the multiplicities in the tensor product of any two finite-dimensional irreducible representations of a connected and simply connected Lie group. Finding their *polyhedral models* is a long-standing problem. The problem asks to express the multiplicity as the number of lattice points in some convex polytope.

Accumulating from the works of Gelfand, Berenstein and Zelevinsky since 1970's, around 1999 Knutson and Tao invented their *hive model* for the type A cases, which led to the solution of the *saturation conjecture*. Outside type A, Berenstein and Zelevinsky's models are still the only known polyhedral models up to now. Those models lose a few nice features of the hive model.

In this talk, I will explain how to use *upper cluster algebras*, an interesting class of commutative algebras introduced by Berenstein-Fomin-Zelevinsky, to discover new polyhedral models for all Dynkin types. Those new models improve the ones of Berenstein-Zelevinsky's, or in some sense generalize the hive model.

It turns out that the quivers of relevant upper cluster algebras are related to the Auslander-Reiten theory of presentations, which can be viewed as a categorification of these quivers. The upper cluster algebras are graded by triple dominant weights, and the dimension of each graded component counts the corresponding tensor multiplicity.

The proof also invokes another categorification – Derksen-Weyman-Zelevinsky's quiver-with-potential model for the cluster algebra. The bases of these upper cluster algebras are parametrized by μ-supported g-vectors. The polytopes will be described via stability conditions.

The talk is based on the preprint arXiv:1603.02521.

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