

# Microlocal brane structures and the classifying map

Xin Jin \*

[xin.jin@bc.edu](mailto:xin.jin@bc.edu)

---

For any Lagrangian  $L$  in the cotangent bundle of a manifold  $X$ , one can define a sheaf of stable infinity-categories using microlocal sheaf theory over ring spectra. When  $L$  is smooth, the sheaf is a locally constant sheaf of categories with fiber equivalent to the category of spectra, and we call it the sheaf of brane structures on  $L$ . The sheaf of brane structures naturally admits a classifying map. We show that the classifying map is homotopic to the composition of the stable Gauss map and the delooping of the  $J$ -homomorphism in stable homotopy theory. The main ingredients include a sheaf quantization of a Hamiltonian  $\coprod_n BO(n)$ -action on the stabilized cotangent bundle of  $R^N$  that is naturally related to Bott periodicity, and the category of correspondences developed by Gaitsgory–Rozenblyum. If time permits, I will talk about how one can define a microlocal sheaf category (and its obstruction) on a Lagrangian skeleton of a Liouville manifold over ring spectra as a consequence of the above result.

---

\*Department of Mathematics, Boston College, Gasson Hall, 140 Commonwealth Avenue, Chestnut Hill, MA 02467, USA