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Super-resolution property of splitting methods for Dirac equation in the nonrelativistic limit regime

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We establish error bounds of the Lie-Trotter splitting and Strang splitting for the Dirac equation in the nonrelativistic limit regime in the absence of external magnetic potentials. In this regime, the solution admits high frequency waves in time. Surprisingly, we find out that the splitting methods exhibit super-resolutions, i.e. the methods can capture the solutions accurately even if the time step size is much larger than the sampled wavelength. Lie splitting shows half order uniform convergence w.r.t temporal wave length. Moreover, if the time step size is non-resonant, Lie splitting would yield an improved uniform first order uniform error bound. In addition, we show Strang splitting is uniformly convergent with half order rate for general time step size and uniformly convergent with three half order rate for non-resonant time step size. Finally, numerical examples are reported to validate our findings.

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