

# Minicourse on statistical reconstruction problems

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## Lecture 1. Reconstruction on trees and unpredictable paths

Suppose a random bit at the root of a finite tree is transmitted to the leaves via binary symmetric channels. When do the leaf values contain significant information on the original bit? [1]

This problem becomes much harder when the channel is not binary. The binary case has applications to constructing unpredictable paths in a lattice, [2, 3] and to community detection in block models.

## Lecture 2. Detecting the trail of a random walk, and a hidden clique in a random graph

Suppose that the vertices of the lattice are endowed with a random scenery. A random walker, starting from the origin, replaces the state of the scenery along its path using another law (independently at each visited vertex). For which laws of the random walk and which dimensions can the resulting scenery be distinguished from the original one? [4, 5]

## Lecture 3. Detecting a missing point in a perturbed lattice

Consider a perturbed lattice obtained by adding independent  $d$ -dimensional Gaussian variables  $Y_v$  to the lattice points in  $d$  dimensions.

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Suppose that one point is removed from this perturbed lattice; is it possible for an observer, who sees just the remaining points, to detect that a point is missing? [7]

## Lecture 4. Trace reconstruction for the deletion channel

Suppose that an unknown string  $x$  of  $n$  bits is observed through the deletion channel; how many independent outputs (traces) of this channel are needed to reconstruct  $x$  with high probability? [8, 9, 10].

## References

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