

Halting problems in abelian networks

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In the abelian sandpile model on the d -dimensional lattice \mathbb{Z}^d , each site that has at least $2d$ particles gives one particle to each of its $2d$ nearest neighbors. An “avalanche” is what happens when you iterate this move. Hannah Cairns proved in <https://arxiv.org/abs/1508.00161> that for $d = 3$, questions of the form “Will this avalanche ever stop?” are algorithmically undecidable: they can be as hard as the halting problem!

This infinite unclimbable peak is surrounded by appealing finite peaks: In dimension 1, sandpile halting is solved but abelian network halting is still open. (An abelian network is a collection of automata whose state transitions and message passing each obey a commutativity condition. Many abelian networks, such as the sandpile and rotor-router models, have equivalent descriptions as particle systems.) In dimension 2, sandpile halting is still wide open, but “ mod 1 harmonic functions” as in <https://arxiv.org/abs/1703.00827> suggest a way forward.

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