

# Problems in first-passage percolation. 2: Geodesics

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We will discuss properties of geodesics in the first-passage percolation model, especially infinite geodesics (infinite paths whose subsegments are geodesics between their endpoints). Long finite geodesics are expected to be relatively straight. In fact, the geodesic from  $y$  to  $nx$  should converge as  $n$  grows to a limiting infinite geodesic, and this geodesic should remain close to the line passing through  $y$  and  $x$  – leading to an infinite number of infinite geodesics from each starting point  $y$ . At least in two dimensions, such “ $x$ -directed” infinite geodesics should coalesce for fixed  $x$ : changing the starting point  $y$  should affect only a finite initial segment of the geodesic. Newman and collaborators proved versions of these statements in the mid-90’s under an unproven curvature assumption on the model’s limit shape. We will discuss that work, progress in the last 15 years toward proving the conjectured behavior under fully general assumptions, and the large number of open problems that remain. We will assume no knowledge of percolation except the content of the previous talk on limit shapes.

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