

Limits of multiplicative inhomogeneous random graphs and Lévy trees

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We consider a model of inhomogeneous random graphs that extend Erdős–Rényi graphs and that shares a close connection with the multiplicative coalescence, as pointed out by Aldous [*Ann. Probab.*, vol. 25, pp. 812–854, 1997]. These models have been studied first by Aldous and Limic [*Electron. J. Probab.*, vol. 3, pp. 1–59, 1998] and their connected components evolve as a multiplicative coalescent: namely, let N be the number of vertices and let w_1, \dots, w_N be a set of positive weights; we independently put an edge between vertices i and j with probability $p_{i,j} = 1 - e^{-w_i w_j / s}$ (in our case, we consider, $s = w_1 + \dots + w_N$).

Our results are the following: we first generate such graphs by an exploration that reduces to a LIFO queue. This point of view allows to code an appropriate spanning tree of the graph thanks to a contour process (and a modified Lukasiewicz path) and to get a simple control on the surplus edges. The spanning tree encompasses most of the metric structure. This construction also allows to embed such graphs into Galton-Watson trees.

This embedding transfers asymptotically into an embedding of the limit objects into a forest of Lévy trees, which allows us to prove a limit theorem and an explicit construction of the limit objects from the excursions of a Lévy-type process. As a consequence of our construction, we give a transparent and explicit condition for the compactness of the limit objects and determine their fractal dimensions. These results extend and complement several previous results that had obtained via model- or regime-specific proofs, for instance: the case of Erdos-Renyi random graphs obtained by Addario-Berry, Goldschmidt and B. [*Probab. Theory Rel. Fields*, vol. 153, pp. 367–406, 2012], the *asymptotic homogeneous* case as studied by Bhamidi, Sen and Wang [*Probab. Theory Rel. Fields*, vol. 169, pp. 565–641, 2017], or the *power-law* case as considered by Bhamidi, Sen and van der Hofstad [*Probab. Theory Rel. Fields*, vol. 170, pp. 387–474, 2018].

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