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Sharp arithmetic transitions for 1D quasiperiodic operators

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A very captivating question in solid state physics is to determine/understand the hierarchical structure of spectral features of operators describing 2D Bloch electrons in perpendicular magnetic fields, as related to the continued fraction expansion of the magnetic flux. In particular, the hierarchical behavior of the eigenfunctions of the almost Mathieu operators, despite significant numerical studies and even a discovery of Bethe Ansatz solutions has remained an important open challenge even at the physics level.

I will present a complete solution of this problem in the exponential sense throughout the entire localization regime. Namely, I will describe the continued fraction driven hierarchy of local maxima, and a universal (also continued fraction expansion dependent) function that determines local behavior of all eigenfunctions around each maximum, thus giving a complete and precise description of the hierarchical structure. In the regime of Diophantine frequencies and phase resonances there is another universal function that governs the behavior around the local maxima, and a reflective-hierarchical structure of those, phenomena not even described in the physics literature. These results lead also to the proof of sharp arithmetic transitions between pure point and singular continuous spectrum, in both frequency and phase, as conjectured since 1994. This part of the talk is based on the papers joint with W. Liu.

Within the singular continuous regime, it is natural to look for further, dimensional transitions. I will present a sharp arithmetic transition result in this regard that holds for the entire class of analytic quasiperiodic potentials, based on the joint work with S. Zhang.

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