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Friable Turán-Kubilius inequality : a survey

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Let $S(x,y)$ denote the set of y-friable integers not exeeding x, i.e. the set of those natural integers $n \leqslant x$ that are free of prime factors exceeding y. Put $\Psi(x,y) := |S(x,y)|$. The friable Turán-Kubilius inequality is an inequality of the type

$$\sup_f \frac{1}{\Psi(x,y)} \sum_{n \leqslant x} |f(n) - \mathbb{E}_f(x,y)|^2 \leqslant C(x,y)\mathbb{V}_f(x,y)$$

where the supremum runs over all complex additive arithmetical functions f and $\mathbb{E}_f(x,y)$, $\mathbb{V}_f(x,y)$ stand respectively for the expectation and variance of a probabilistic model for f over $S(x,y)$. In connection with Kubilius' model, the classical case $y = x$ has been extensively studied in the literature.

We shall describe results and methods developed in the last 35 years for tackling this problem, having led to asymptotic formulae for $C(x,y)$ whenever x and y tend to infinity.

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