Programme sur les probabilités en théorie des nombres
21 mai au 1er juin 2018

Programme on Probability in Number Theory
May 21 - June 1, 2018

Primitive sets

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A set of positive integers is *primitive* if no member divides another. This concept was important for the elementary proof of Erdős that the abundant numbers have a natural density. There is another early connection in the work of Besicovitch who showed that while the lower density of a primitive set is always zero, the upper density can be arbitrarily close to 1/2. This work involved the density of numbers with a divisor in a given dyadic interval, a subject further developed by Erdős, Tenenbaum, Ford, and Koukoulopoulos.

Erdős showed in 1935 that if A is a primitive set not containing 1, then the sum of $1/(a \log a)$ with $a \in A$ is not only finite, but universally bounded over all A. He asked in a seminar in Limoges in the 1980s if this universal bound is attained by the set of primes. There has been progress towards this by Erdős and Zhang, Banks and Martin, Clark, and others. In this talk I will discuss some recent developments, including some connections with prime number "races".

This is a joint work with Jared D. Lichtmang.

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