

Probabilist questions raised by two number theoretic topics

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The first question is related to the Erdős-Rényi probabilist model for sums of *pseudo s -th powers*. The problem here is to manage the sum of random variables which are close from being independent, but not completely... A very useful tool for dealing with such a situation is the *Janson inequality*, but... in some instances, this inequality induces a term which is of the same order of magnitude as the main term, although this seems to be an artefact: can one shortcut it?

The second question is connected with a probabilist theme called *transportation*, although we are interested in a sort of *worst transportation*, opposed to the more classical *optimal transportation*, namely

Question. Let ρ_1 and ρ_2 be two bounded real probability distributions. Find $\Delta = \Delta(\rho_1, \rho_2)$ as small as possible such that for any pair of random variables X_1, X_2 on a probability space (Ω, \mathcal{F}, P) with respective densities ρ_1 and ρ_2 , there exists ω in Ω such that $|X_1(\omega) - X_2(\omega)| \leq \Delta$.

We shall show how this question naturally arises in the study of the difference between the sum of digits of an integer in two different bases.

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