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Sharp asymptotic profiles for singular solutions to an elliptic equation with a sign-changing nonlinearity

Frédéric Robert *

frederic.robert@univ-lorraine.fr

We analyse the behavior at 0 of positive solutions of

$$-\Delta u = \alpha \frac{u^{2^\star(s)-1}}{|x|^s} - \beta u^q \text{ in } B \setminus \{0\}$$

where $B \subset \mathbb{R}^n$ is the unit ball and $2^\star(s) = 2(n-s)/(n-2)$, $0 < s < 2$. When $\alpha > 0$ and $\beta = 0$, the potential behaviors are classified in the spirit of the pioneer work of Caffarelli-Gidas-Spruck. Conversely, when $\alpha = 0$ and $\beta > 0$, the analysis is also classical (see for instance Véron). In the present talk, I will discuss the case when $\alpha > 0$ and $\beta > 0$. In addition to the classical profiles above when one of the parameters α, β vanishes, we show that the behavior at zero is also governed by two other original profiles. All these profiles but one are *d*ifferential in the sense that the behavior is modeled on solutions to some limiting PDEs. Surprisingly, one of them is *n*on-differential, in the sense that it is modeled on the equation $0 = \alpha \frac{u^{2^\star(s)-1}}{|x|^s} - \beta u^q$. This analysis permits to exhibit examples of all these profiles.

[1] F.C. Cîrstea and F. Robert, Sharp asymptotic profiles for singular solutions to an elliptic equation with a sign-changing non-linearity. Proc. Lond. Math. Soc. (3) 114 (2017), no. 1, 1-34.

[2] F.C. Cîrstea, F. Robert and J. Vétois, Examples of sharp asymptotic profiles of singular solutions to an elliptic equation with a sign-changing non-linearity, preprint (2017).

This is a series of joint works with Florica Cîrstea (Sydney) and Jérôme Vétois (Montréal).

*Institut Élie Cartan de Lorraine, Université de Lorraine, BP 70239, F-54506 Vandoeuvre-lès-Nancy Cedex, FRANCE