

# Notes on a non-equilibrium stationary state for a stochastic Klein-Gordon equation

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We consider the problem of non-equilibrium stationary states for a stochastic one-dimensional periodic Klein-Gordon equation having finite-dimensional Brownian motion driving terms and finite-dimensional dissipation terms. The model is akin to those considered by Eckmann, Pillet, and Rey-Bellet, but with oscillator chains replaced by a continuous field. For the linear problem, the Fourier-like modes of the field have uniformly bounded variances with respect to the stationary state, reminiscent of equipartition of energy for the equilibrium case; this boundedness implies a certain regularity of the field. The problems of existence and regularity of a stationary state for the nonlinear problem remain open. Here we present partial results concerning mode covariances in the setting where the non-linearity, a function of the field, is uniformly Lipschitz.

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