

# Uncertainty quantification and thermodynamic formalism

Luc Rey-Bellet \*

[luc@math.umass.edu](mailto:luc@math.umass.edu)

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Consider a baseline probabilistic model  $P$  (think of the model one actually computes or calculates with) which is viewed, due to error and/or approximations, as the imperfect representation of the “true” model  $Q$ . One also postulates that the true model  $Q$  is at “distance”  $d$  from the baseline model  $P$  and that one is interested in performing some measurement with  $P$  (say compute an expectation or a variance, or compute the probability of a rare event). Can one obtain (tight and computable) performance guarantees for the possible values of the measurement under the true model  $Q$ ? This question is a basic problem in uncertainty quantification (model form uncertainty).

We show that by using the relative entropy and the relative Renyi entropy, variational principles, and concentration inequalities, one can provide such tight and computable bounds which vastly improve on classical information inequalities (such as Pinsker inequality). Moreover our new information inequalities scale properly with time and/or the size of the systems allowing to perform uncertainty quantification for steady states of Markov processes and for phase diagrams. Our results also naturally extend to quantum systems.

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\*Mathematics and Statistics, University of Massachusetts Amherst, 710 N Pleasant St, Amherst, MA MA 01002, USA