A tale of two triangles

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We will discuss two triangles analogous to Pascal’s triangle (or the arithmetic triangle). The first is a multiplicative analogue of Pascal’s triangle, and the second is a slight modification of Stern’s diatomic array. A typical result is the following: let

\[ \prod_{i=0}^{n-1} (1 + x^{2^i} + x^{2^i+1}) = \sum a_j x^j. \]

Then

\[ \sum a_j^2 = 3 \cdot 7^{n-1}. \]

The results related to Stern’s diatomic array can be vastly generalized, and some special cases are of particular interest.

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