

A formal proof of the Littlewood-Richardson Rule

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The Littlewood-Richardson is a rule which give a way to compute the coefficients of the expansion in the Schur basis of the product of two Schur functions. It is a very important result with applications ranging from computer algebra to group theory and quantum physics and even possibly complexity theory. The coefficient are expressed by counting some particular kind of skew tableaux. The algorithm enumerating those objects and counting them is called the Littlewood-Richardson Rule.

The Littlewood-Richardson rule is notorious for the number of errors that appeared prior to its complete, published proof. Several published attempts to prove it are incomplete, and it is particularly difficult to avoid errors when doing hand calculations with it: even the original example in D. E. Littlewood and A. R. Richardson (1934) contains an error. The first rigorous proofs of the rule were given four decades after it was found, by Schützenberger (1977). Though there is nowadays no doubt that the rule is correct, We feel that It makes sense to see if computers can help checking that Schützenberger proof was correct.

This talk will be divided in two part: first we will introduce formal proofs and proof assistants. For my proof we are using Coq with the Mathematical Component extension. In the second part, we will present the proof itself, and conclude by explaining the difficulties and challenge of developing such a proof, and giving a idea of the amount of work needed to check a new theorem.

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