

# Quantum entropic ambiguities and Tomita-Takesaki theory

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Given an algebra of observables  $\mathcal{A}$  and a state  $\omega$ , a density matrix  $\rho_\omega$  acting on  $\mathcal{H}_\omega$  can be obtained through the GNS construction  $(\mathcal{H}_\omega, \pi_\omega)$  that gives rise to the same expectation values as  $\omega$  for elements  $a \in \mathcal{A}$ , that is,  $\omega(a) \equiv \text{Tr}_{\mathcal{H}_\omega}(\rho_\omega \pi_\omega(a))$ . An entropy can be assigned to the state  $\omega$  by computing the von Neumann entropy of the density matrix  $S(\rho_\omega) = -\text{Tr}_{\mathcal{H}_\omega}(\rho_\omega \log \rho_\omega)$ . This has proved to be useful, e.g., in the study of entanglement properties of identical particles (Balachandran et al., Phys. Rev. Lett. 110, 080503 (2013)). However, there are situations for which this density matrix is not unique, thus leading to an entropy ambiguity. This occurs whenever the irreducible components of the representation  $\pi_\omega$  appear in  $\mathcal{H}_\omega$  with multiplicities (Balachandran et al., Eur. Phys. J. Plus (2013)). In the present work, we develop an interpretation of this phenomenon as a gauge symmetry arising from the action of unitaries in the commutant of the representation via Tomita-Takesaki modular theory. In the finite-dimensional case, a complete characterization of the ambiguity can be given in terms of the modular data, and a physical interpretation can be obtained in terms of an equivalent description of the system as a bipartite system. We extend our analysis to the case of group transformation  $C^*$ -algebras describing the algebras of observables of quantum systems obtained by quantization on configuration spaces of the form  $G/H$ , with  $G$  a compact Lie group and  $H$  a finite, non-abelian subgroup.

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