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Quantum Hurwitz numbers and their asymptotics

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Amongst the various weight generating functions that may be chosen in defining weighted Hurwitz numbers, a particularly interesting one is the (exponential of) the quantum dilogarithm (or, equivalently, the first factor in the Jacobi triple product formula for theta functions). It turns out that this gives, for real q lying between 0 and 1, a real, positive, normalizable probability measure which coincides with the distribution of a quantum bosonic gas with linear energy spectrum, if the energy associated with any branching structure is chosen as proportional to the colength (i.e. the degree of degeneracy) of the partition describing the ramification structure. Two sorts of asymptotics for such "quantum Hurwitz numbers" are considered: semi-classical, where the parameter playing the role of Planck's constant tends to zero ($q \to 1$) and the zero temperature limit ($q \to 0$). In the first limit, the quantum weight tends to the Dirac measure supported on simple partitions - resulting in the "simple"(double or single) Hurwitz numbers considered by Okounkov and Pandharipande and others. In the second limit, the weight tends to a (normalized) Dirac measure supported on configurations consisting of one weighted branch point plus two unweighted ones - hence, the case of Belyi curves (or, equivalently, strictly monotonic paths in the Cayley graph generated by transpositions). The τ-function that serves as generating function for the quantum Hurwitz numbers tends, in these two limits, to the ones for simple branchings and the Belyi curve (strictly monotone) case.

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