FRT PRESENTATION
of the
LOOP ALGEBRA
and
APPLICATIONS

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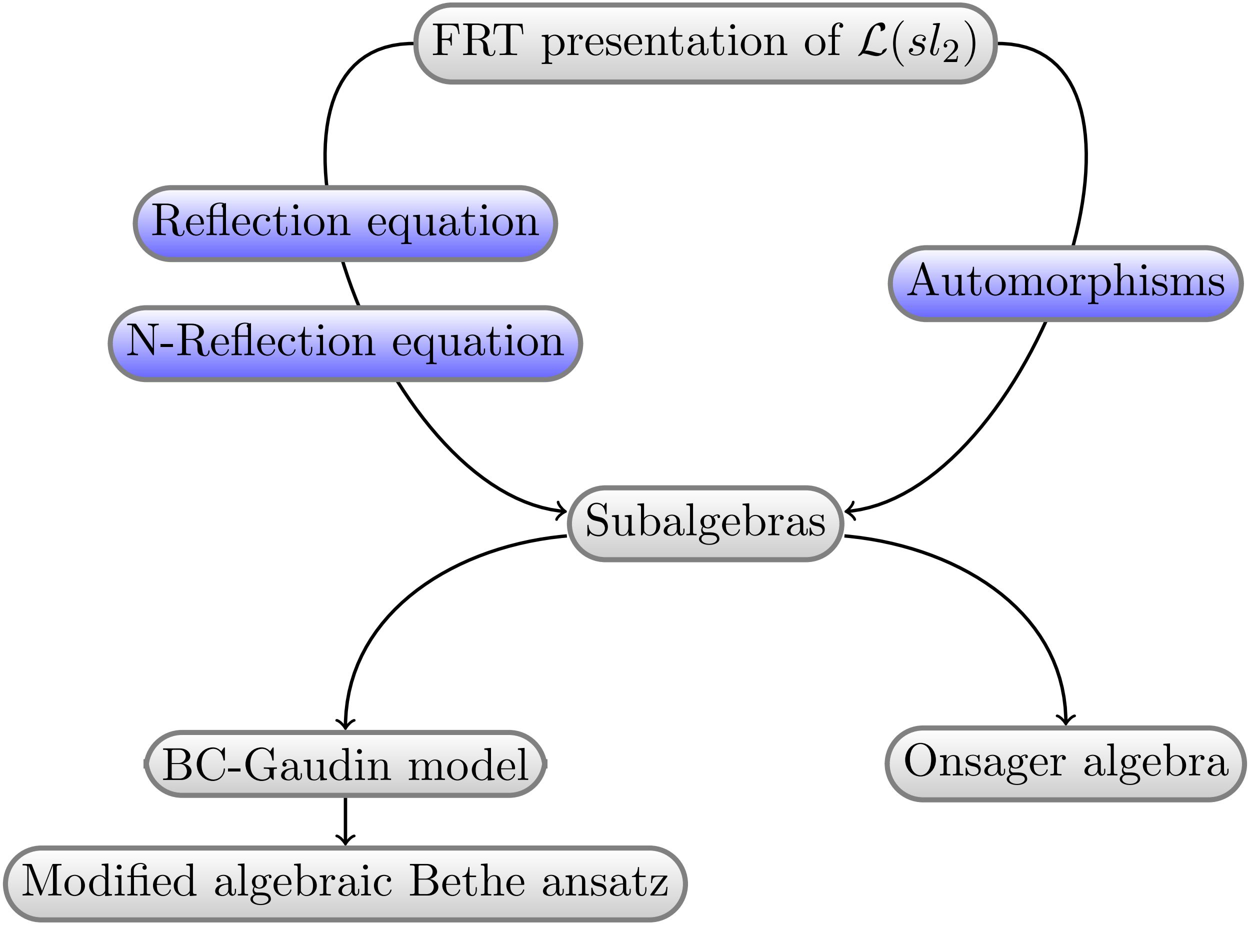
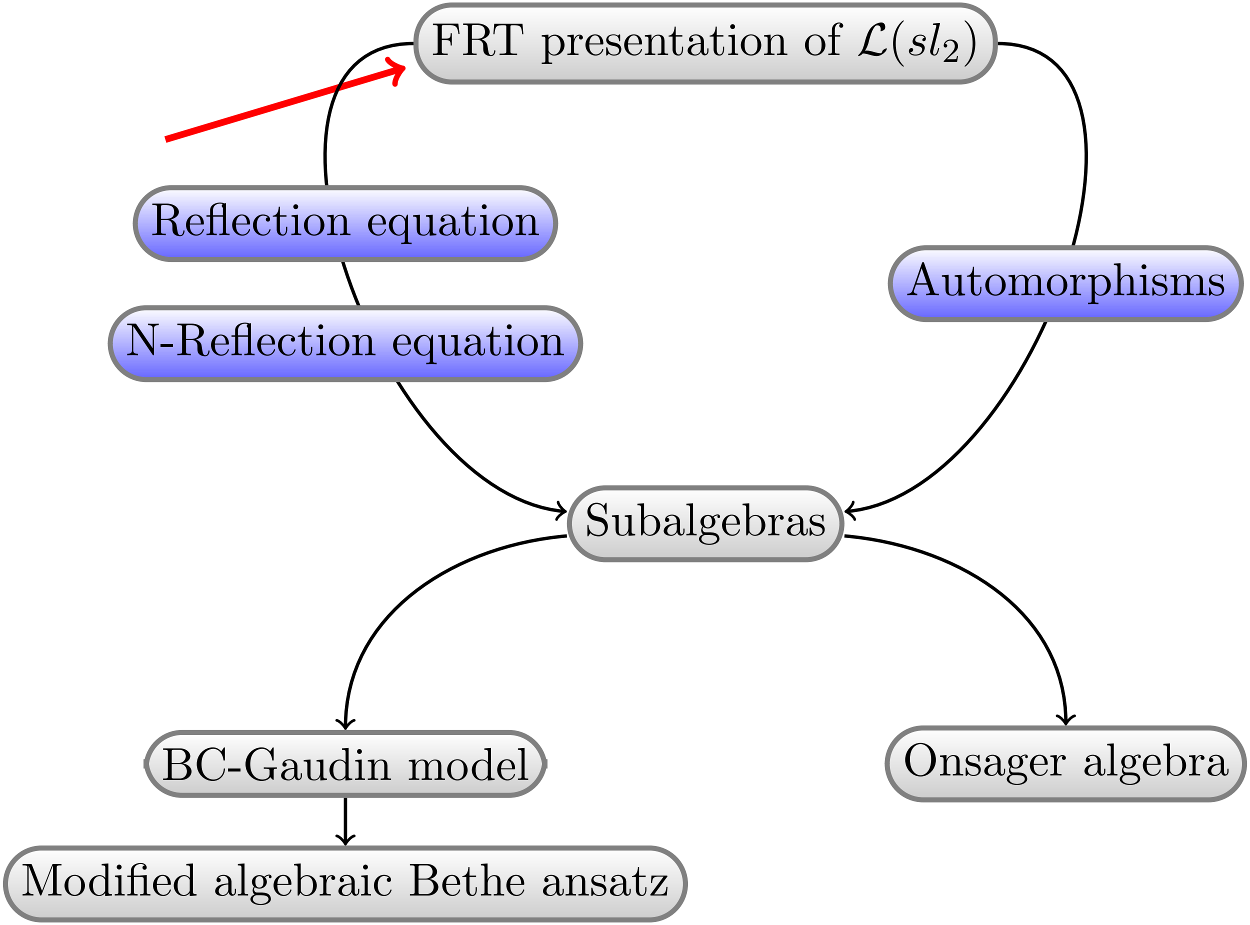

FRT presentation of $\mathcal{L}(sl_2)$
Reflection equation
N-Reflection equation
Automorphisms
Subalgebras
BC-Gaudin model
Onsager algebra
Modified algebraic Bethe ansatz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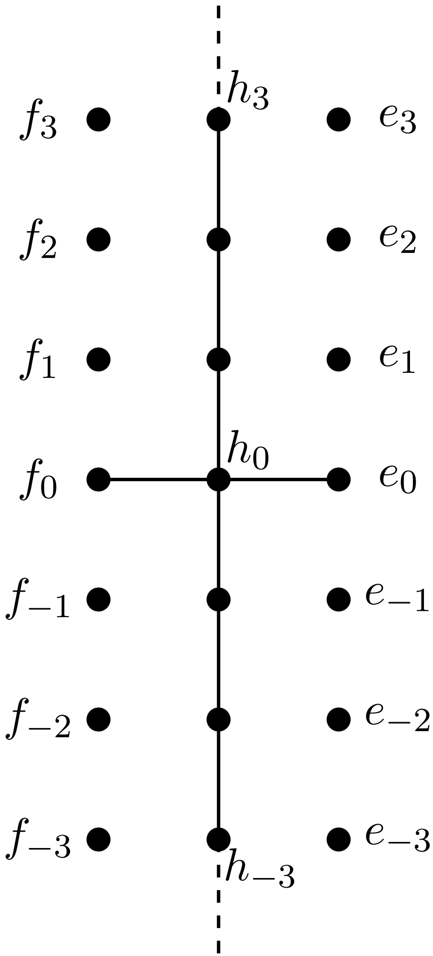
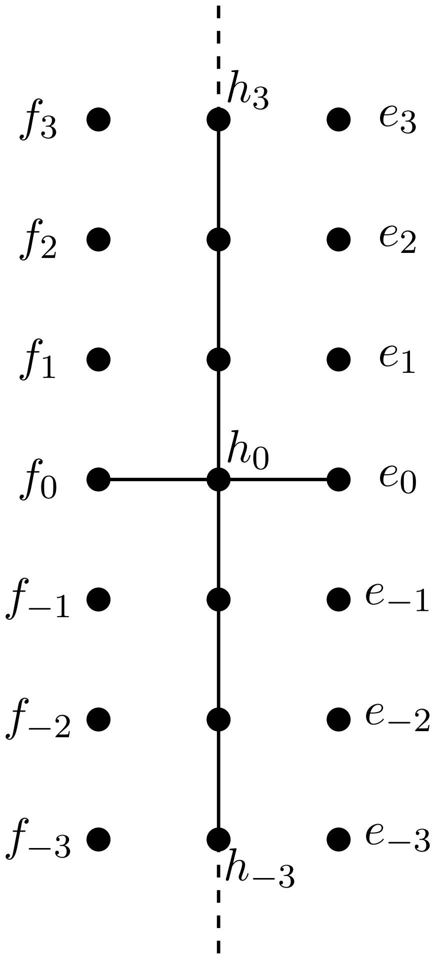
Loop algebra of $sl_2 \sim \mathcal{L}(sl_2)$



Generators

$$h_n,\; e_n,\; f_n \qquad \text{for} \quad n \in \mathbb{Z}$$



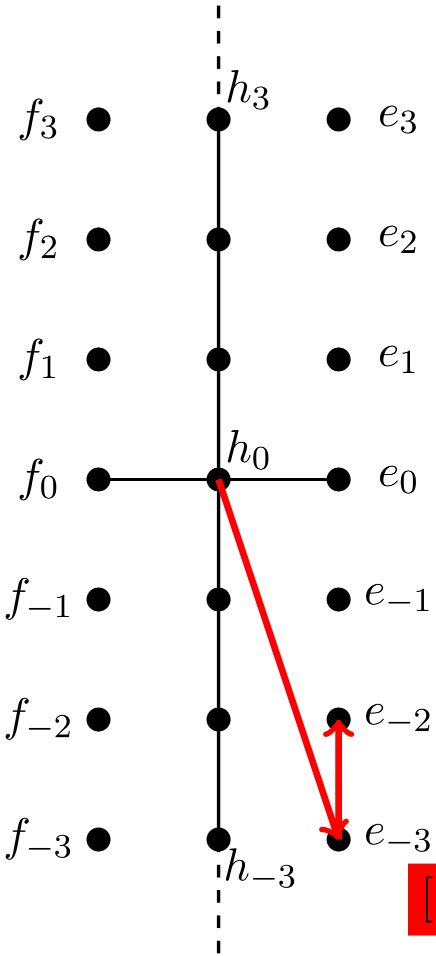
Generators

$h_n,\ e_n,\ f_n \qquad \text{for} \quad n \in \mathbb{Z}$

Defining relations of $\mathcal{L}(sl_2)$

$$[h_i, e_j] = 2e_{i+j}$$
$$[h_i, f_j] = -2f_{i+j}$$
$$[e_i, f_j] = h_{i+j}$$

Loop algebra of sl_2 $\sim$ $\mathcal{L}(sl_2)$



$[h_1, e_{-3}] = 2e_{-2}$

Generators

$h_n, \ e_n, \ f_n$ for $n \in \mathbb{Z}$

Defining relations of $\mathcal{L}(sl_2)$

$$[h_i, e_j] = 2e_{i+j}$$
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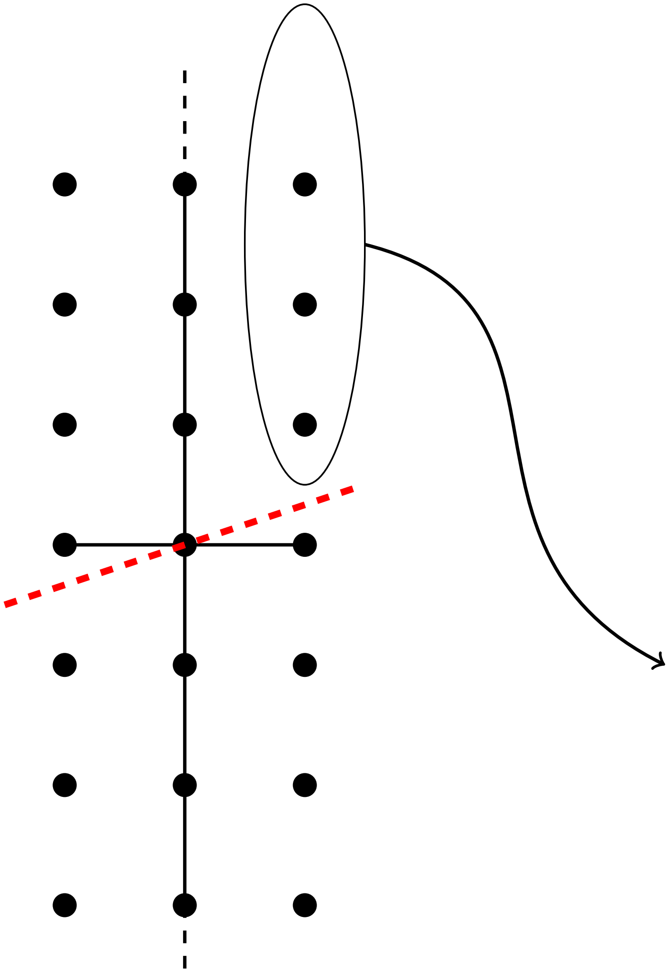
Current presentation



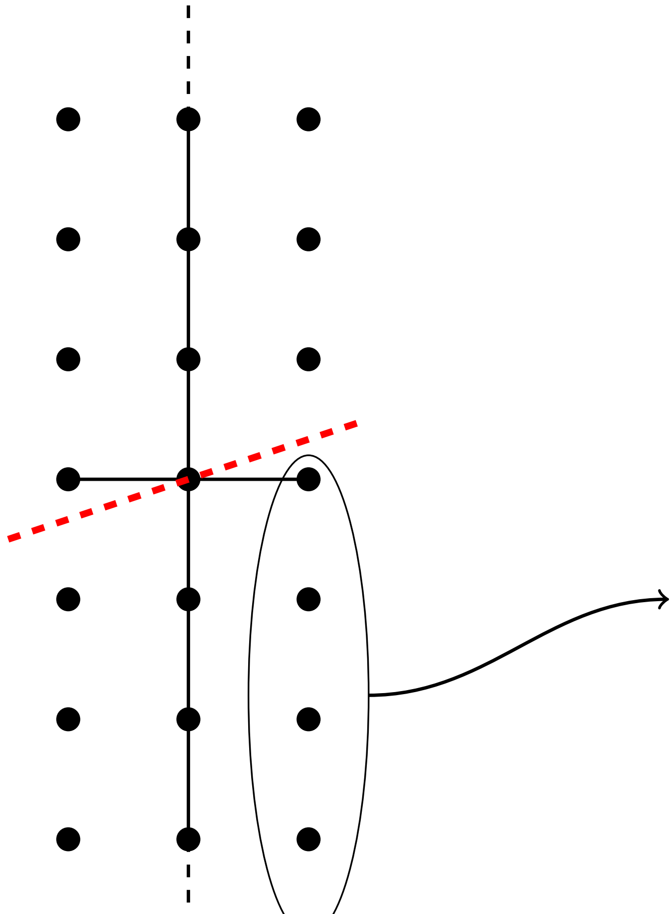
Positive currents

$$H^+(x) = \frac{h_0}{2} + \sum_{k=1}^{\infty} x^k \; h_k$$

$$F^+(x) = 2\sum_{k=0}^{\infty} x^k \; f_k$$

$$E^+(x) = 2\sum_{k=1}^{\infty} x^k \; e_k$$

Current presentation



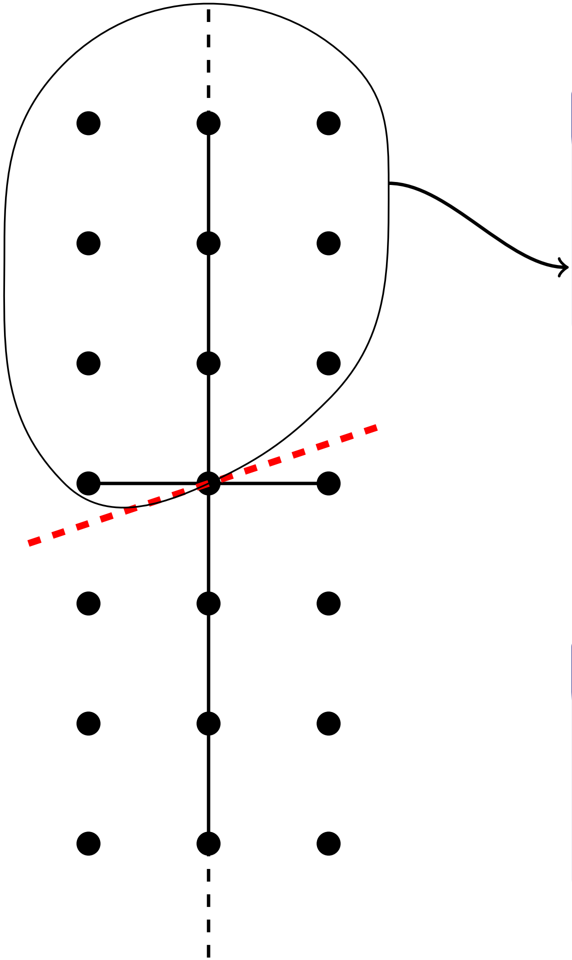
Negative currents

$$H^-(x) = \frac{h_0}{2} + \sum_{k=1}^{\infty} \frac{1}{x^k}\, h_{-k}$$

$$F^-(x) = 2\sum_{k=1}^{\infty} \frac{1}{x^k}\, f_{-k}$$

$$E^-(x) = 2\sum_{k=0}^{\infty} \frac{1}{x^k}\, e_{-k}$$

FRT presentation



Positive current

$$T^+(x) = \begin{pmatrix} H^+(x) & F^+(x) \\ E^+(x) & -H^+(x) \end{pmatrix}$$

Negative current

$$T^-(x) = -\begin{pmatrix} H^-(x) & F^-(x) \\ E^-(x) & -H^-(x) \end{pmatrix}$$

FRT presentation

Proposition

$\mathcal{L}(sl_2)$ is presented equivalently by

$$[\ T_1^{\pm}(x)\ ,\ T_2^{\pm}(y)\ ] \ = \ [\ T_1^{\pm}(x) + T_2^{\pm}(y)\ ,\ r_{12}(x/y)\ ]$$
$$[\ T_1^{+}(x)\ ,\ T_2^{-}(y)\ ] \ = \ [\ T_1^{+}(x) + T_2^{-}(y)\ ,\ r_{12}(x/y)\ ]$$

where

$$r(x) = \frac{1}{x-1}\begin{pmatrix} -\frac{1}{2}(x+1) & 0 & 0 & 0 \\ 0 & \frac{1}{2}(x+1) & -2 & 0 \\ 0 & -2x & \frac{1}{2}(x+1) & 0 \\ 0 & 0 & 0 & -\frac{1}{2}(x+1) \end{pmatrix}$$

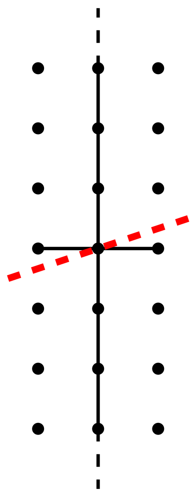
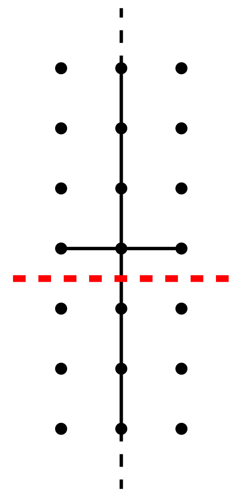
FRT presentation

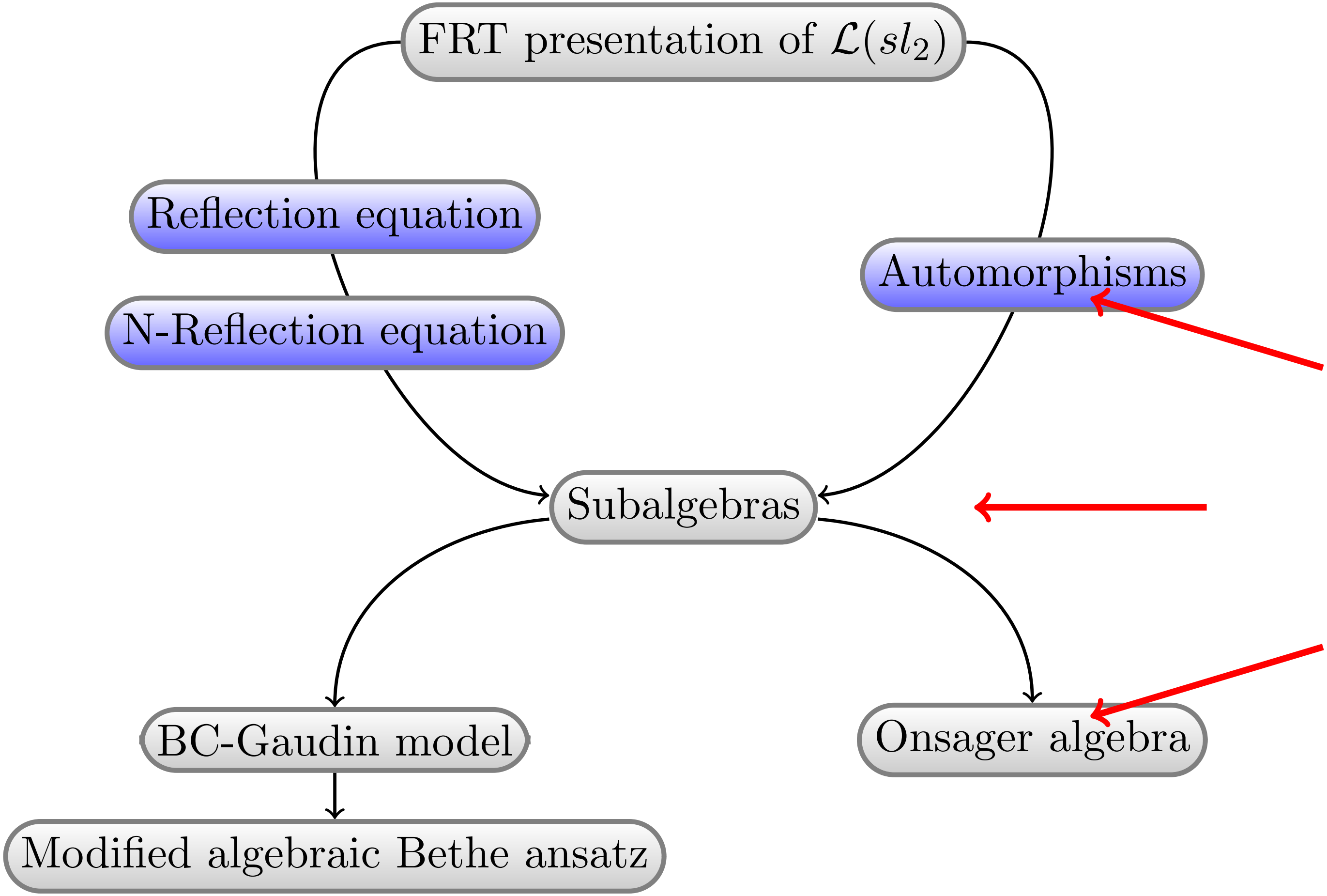
- r is called *classical r-matrix*, satisfies the classical Yang-Baxter equation:

  $$[\, r_{13}(x_1/x_3)\ ,\ r_{23}(x_2/x_3)\,] = [\, r_{13}(x_1/x_3) + r_{23}(x_2/x_3)\ ,\ r_{12}(x_1/x_2)\,]$$

  and is skew-symmetric

  $$r_{12}(x) = -r_{21}(1/x)$$

-  $\longrightarrow$ Trigo  $\longrightarrow$ Rational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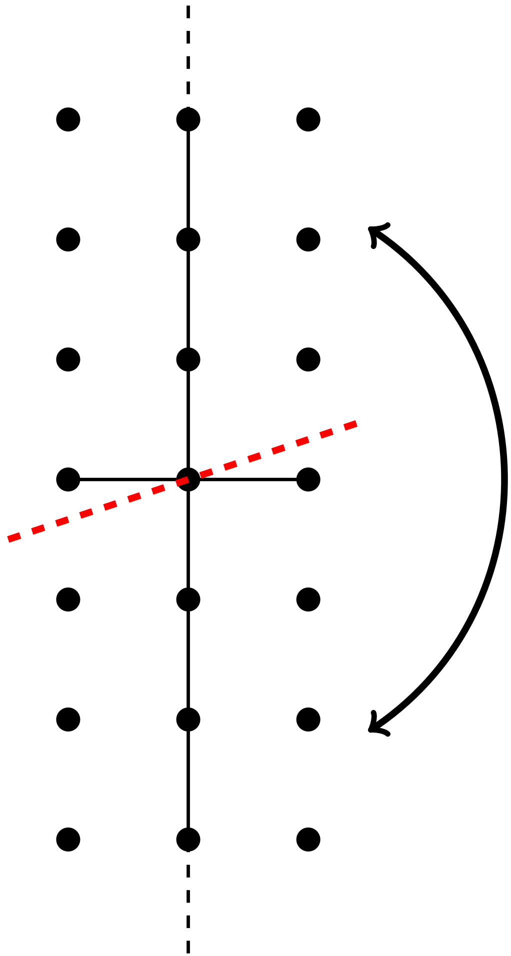
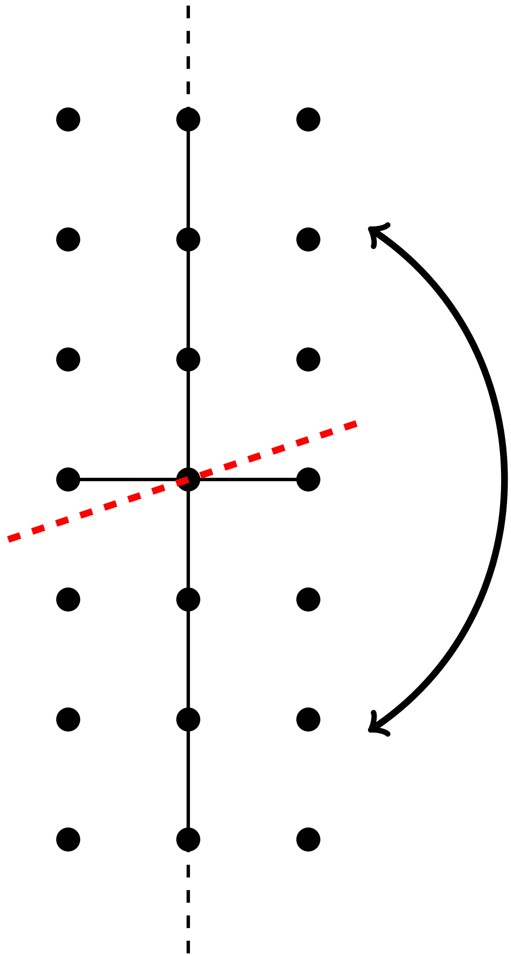
Automorphisms



Automorphisms



Automorphisms of $\mathcal{L}(sl_2)$

$$T^{\pm}(x) \mapsto U(x)\ T^{\mp}(1/x)^t\ U(x)^{-1}$$

if

$$\Big[\ U_1(x)U_2(y)\ ,\ r_{12}(x/y)\ \Big] = 0$$

Fixed point subalgebra of $\mathcal{L}(sl_2)$

$$B(x) = T^+(x) + U(x)\ T^-(1/x)^t\ U(x)^{-1}$$

is fixed by the automorphism and satisfies

$$[\ B_1(x)\ ,\ B_2(y)\ ] = [\ \overline{r}_{21}(y,x)\ ,\ B_1(x)\ ] + [\ B_2(y)\ ,\ \overline{r}_{12}(x,y)\ ] \quad (re)$$

where

$$\overline{r}_{12}(x,y) = r_{12}(x/y) + U_1(x)\ r_{12}^{t_1}(1/(xy))\ U_1(x)^{-1}$$

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- $\overline{r}$ satisfies non-standard classical Yang-Baxter equation
- (re) is a limit of the reflection equation
- (re) is the FRT presentation of the subalgebra generated by $B(x)$
- There exist two Abelian subalgebras

Fixed point subalgebra of $\mathcal{L}(sl_2)$

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Gaudin type Abelian subalgebra

$$t(x) = tr B(x)^2$$

satisfies

$$[t(x)\ ,\ t(y)] = 0$$

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$$\overline{r}_{12}(x,y) = r_{12}(x/y) + U_1(x)\ r_{12}^{t_1}(1/(xy))\ U_1(x)^{-1}$$

Ising type Abelian subalgebra

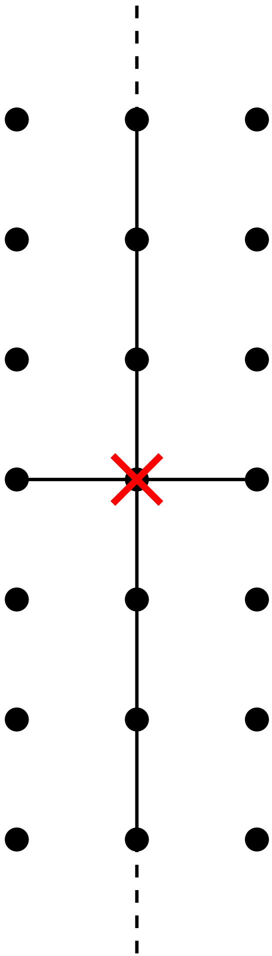
If $M(x)$ satisfies

$$\Big[\ tr_1(\overline{r}_{12}(x,y)M_1(x))\ ,\ M_2(y)\ \Big] = 0$$

then $$b(x) = tr M(x)B(x)$$

satisfies $$[b(x)\ ,\ b(y)] = 0$$

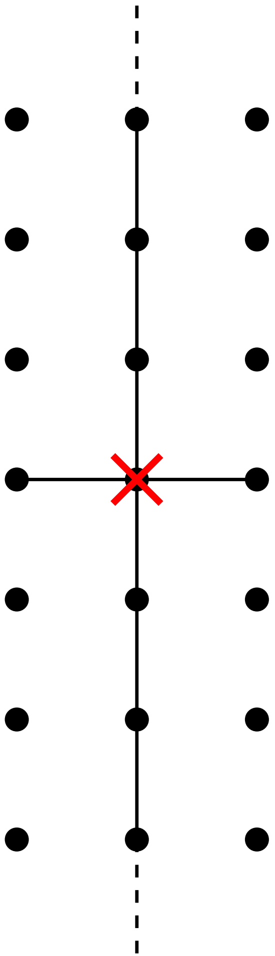
Example: Onsager algebra



$$U = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} e_n &\mapsto f_{-n} \\ f_n &\mapsto e_{-n} \\ h_n &\mapsto -h_{-n} \end{aligned}$$

Example: Onsager algebra



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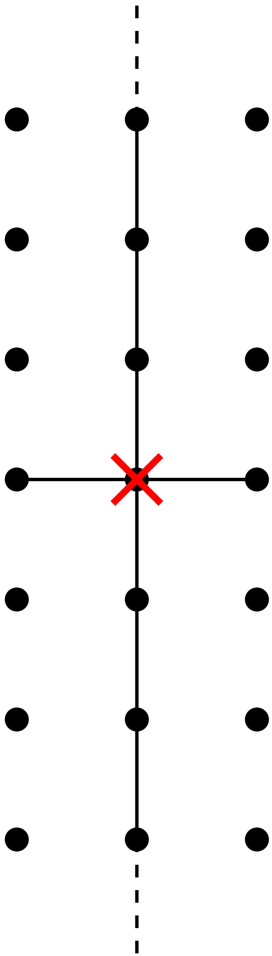
Proposition

(re) provides the FRT presentation of the Onsager algebra

Example: Onsager algebra



$$U = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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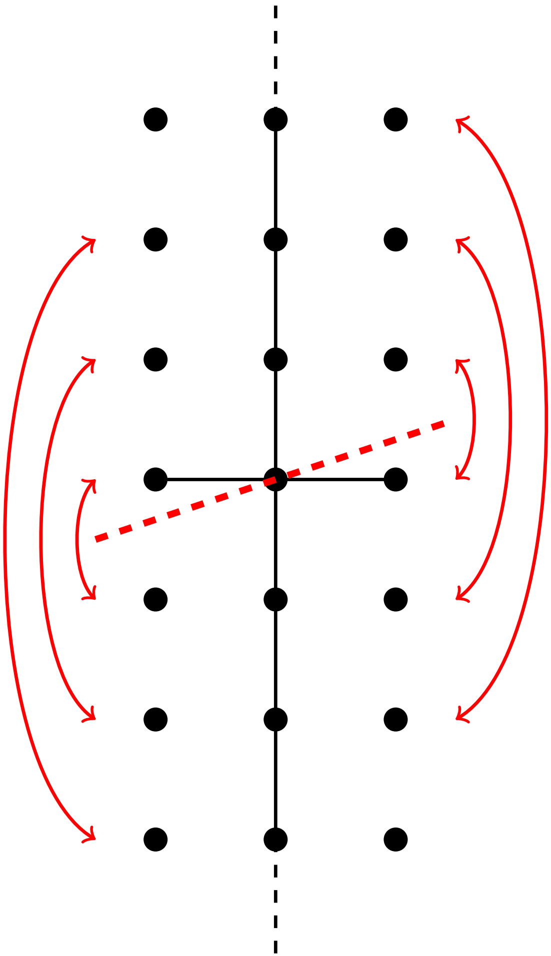
Conserved charges

$b(x) = tr M(x) B(x)$ gives the conserved charges commuting with the Ising model

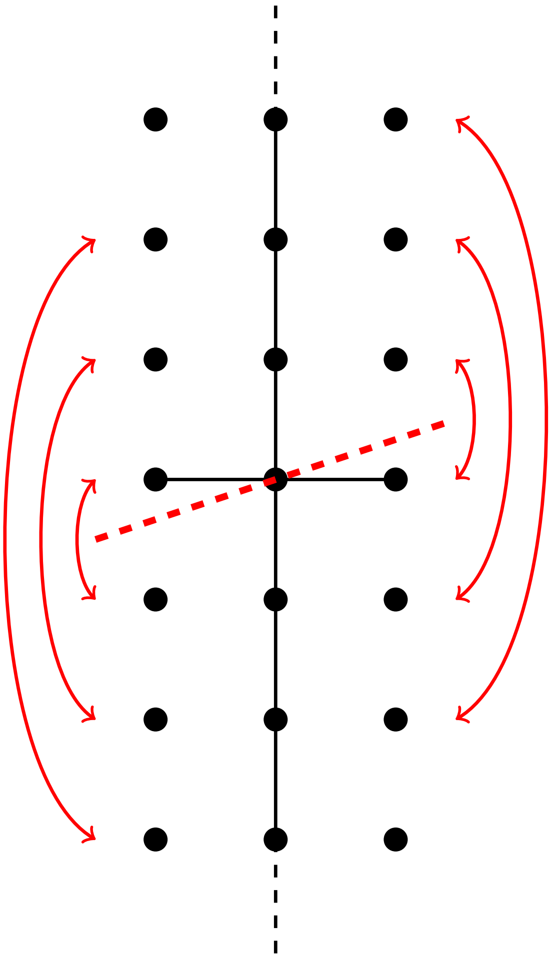
Example: augmented Onsager algebra



$$U(x) = \begin{pmatrix} 0 & 1/\sqrt{x} \\ -\sqrt{x} & 0 \end{pmatrix}$$

$$\begin{array}{ccc} e_n & \mapsto & e_{-n+1} \\ f_n & \mapsto & f_{-n-1} \\ h_n & \mapsto & h_{-n} \end{array}$$

Example: augmented Onsager algebra



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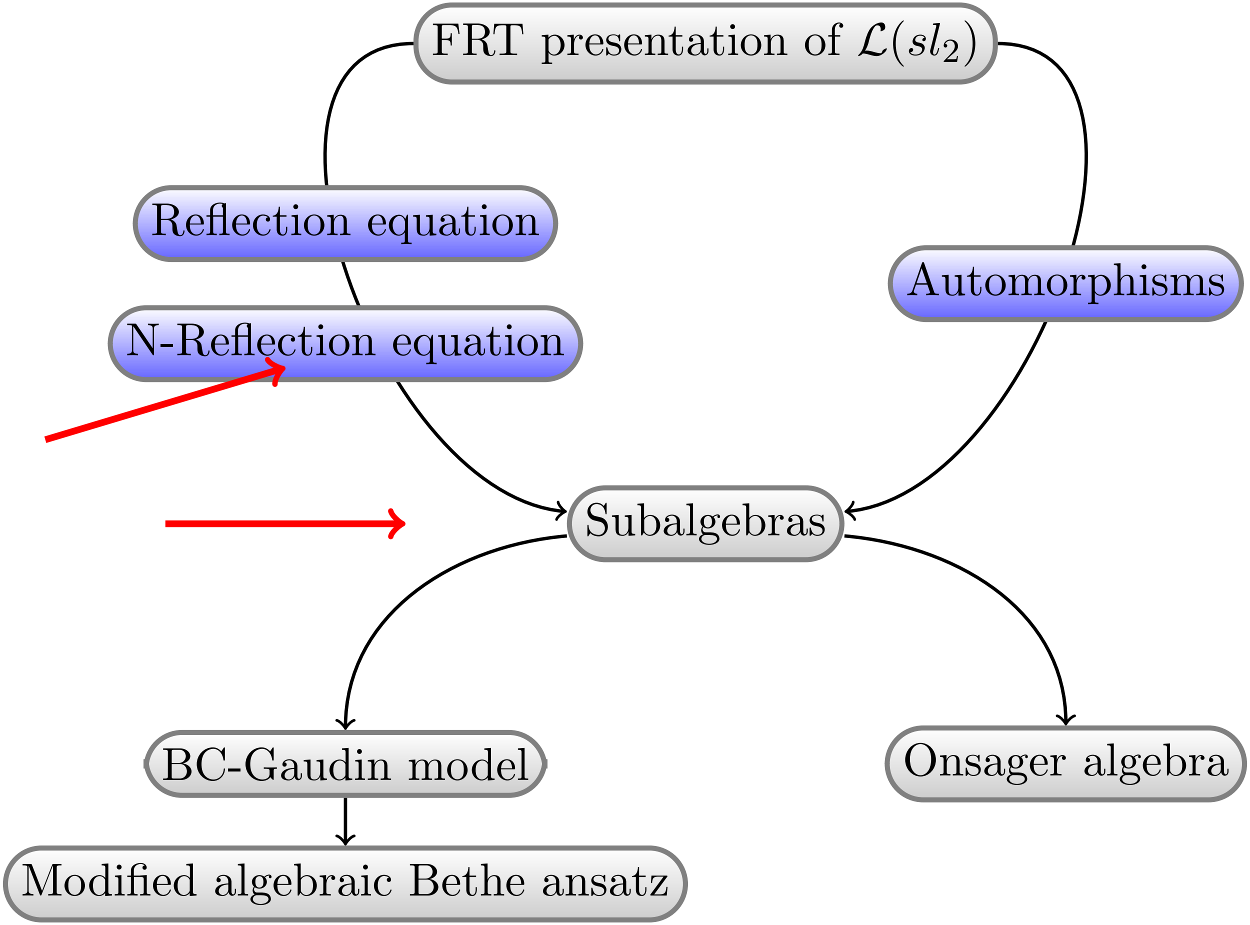
Proposition

(re) provides the FRT presentation of the augmented Onsager algebra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Reflection equation

Goal

Construction of subalgebras

Reflection equation

Goal

Construction of subalgebras

Idea

We want to find constraints on $k(x)$ such that

$$B(x) = T^{+}(x) - k(x)\ T^{-}(1/x)\ k(x)^{-1}$$

satisfies

$$[\ B_1(x)\ ,\ B_2(y)\ ] = [\ \overline{r}_{21}(y,x)\ ,\ B_1(x)\ ] + [\ B_2(y)\ ,\ \overline{r}_{12}(x,y)\ ]$$

for some $\overline{r}$

Reflection equation

Classical reflection equation

$$r_{12}(x/y)\ k_1(x)k_2(y) \quad + \quad k_1(x)\ r_{21}(xy)\ k_2(y)$$
$$=$$
$$k_2(y)\ r_{12}(xy)\ k_1(x) \quad + \quad k_1(x)k_2(y)\ r_{21}(x/y)$$

Reflection equation

Classical reflection equation

$$r_{12}(x/y)\ k_1(x)k_2(y) \quad + \quad k_1(x)\ r_{21}(xy)\ k_2(y)$$
$$=$$
$$k_2(y)\ r_{12}(xy)\ k_1(x) \quad + \quad k_1(x)k_2(y)\ r_{21}(x/y)$$

Solution for $\mathcal{L}(sl_2)$

$$k(x) = \begin{pmatrix} \beta + \gamma/x & -\frac{\alpha(\beta+\rho)}{2}(x - 1/x) \\ \frac{\beta-\rho}{2\alpha}(x - 1/x) & \beta + \gamma x \end{pmatrix}$$

Reflection equation

Classical reflection equation

$$
\begin{array}{ccc}
r_{12}(x/y)\ k_1(x)k_2(y) & + & k_1(x)\ r_{21}(xy)\ k_2(y) \\
 & = & \\
k_2(y)\ r_{12}(xy)\ k_1(x) & + & k_1(x)k_2(y)\ r_{21}(x/y)
\end{array}
$$

Solution for $\mathcal{L}(sl_2)$

$$
k(x) = \begin{pmatrix} \beta + \gamma/x & -\frac{\alpha(\beta+\rho)}{2}(x - 1/x) \\ \frac{\beta-\rho}{2\alpha}(x - 1/x) & \beta + \gamma x \end{pmatrix}
$$

Remark

Subalgebra depending on 4 parameters generalizing Onsager algebra and augmented Onsager algebra.

N-Reflection equation

Remark

Previous constructions are based on involution

N-Reflection equation

Remark

Previous constructions are based on involution

Goal

Construct subalgebras based on automorphisms of order N

N-Reflection equation

Remark

Previous constructions are based on involution

Goal

Construct subalgebras based on automorphisms of order N

We want that

$$\begin{aligned} B(x) &= T(x) \\ &+ g^{(1)}(x)\, k^{(1)}(x)\, T(\tau(x))\, k^{(1)}(x)^{-1} \\ &+ \dots \\ &+ g^{(N-1)}(x)\, k^{(N-1)}(x)\, T(\tau^{N-1}(x))\, k^{(N-1)}(x)^{-1} \end{aligned}$$

satisfies

$$[\, B_1(x)\ ,\ B_2(y)\,] = [\, \overline{r}_{21}(y,x)\ ,\ B_1(x)\,] + [\, B_2(y)\ ,\ \overline{r}_{12}(x,y)\,]$$

N-Reflection equation

Proposition

$$\begin{aligned}
\Big[\quad & r_{12}(x/y) \\
+ \quad & g^{(1)}(y) \quad k_2^{(1)}(y) \quad r_{12}(x/\tau(y)) \quad k_2^{(1)}(y)^{-1} \\
+ \quad & \dots \\
+ \quad & g^{(N-1)}(y)\ k_2^{(N-1)}(y)\ r_{12}(x/\tau^{N-1}(y))\ k_2^{(N-1)}(y)^{-1} \Big] \ k_1(x) \\
& = \\
k_1(x) \Big[\quad & r_{12}(\tau(x)/y) \\
+ \quad & g^{(1)}(y) \quad k_2^{(1)}(y) \quad r_{12}(\tau(x)/\tau(y)) \quad k_2^{(1)}(y)^{-1} \\
+ \quad & \dots \\
+ \quad & g^{(N-1)}(y)\ k_2^{(N-1)}(y)\ r_{12}(\tau(x)/\tau^{N-1}(y))\ k_2^{(N-1)}(y)^{-1} \Big]
\end{aligned}$$

N-Reflection equation

Solution for $\mathcal{L}(sl_2)$ of the 2-reflection equation

$$\tau(x) = \frac{ax+b}{cx-a} \quad , \qquad g^{(1)}(x) = -\frac{(a^2-bc)x}{(cx-a)(ax+b)}$$

$$k^{(1)}(x) = \begin{pmatrix} \tau(x) & 0 \\ 0 & x \end{pmatrix}$$

Solution for $\mathcal{L}(sl_2)$ of the 2-reflection equation

$$\tau(x) = \frac{ax+b}{cx-a} \quad , \qquad g^{(1)}(x) = -\frac{(a^2-bc)x}{(cx-a)(ax+b)}$$

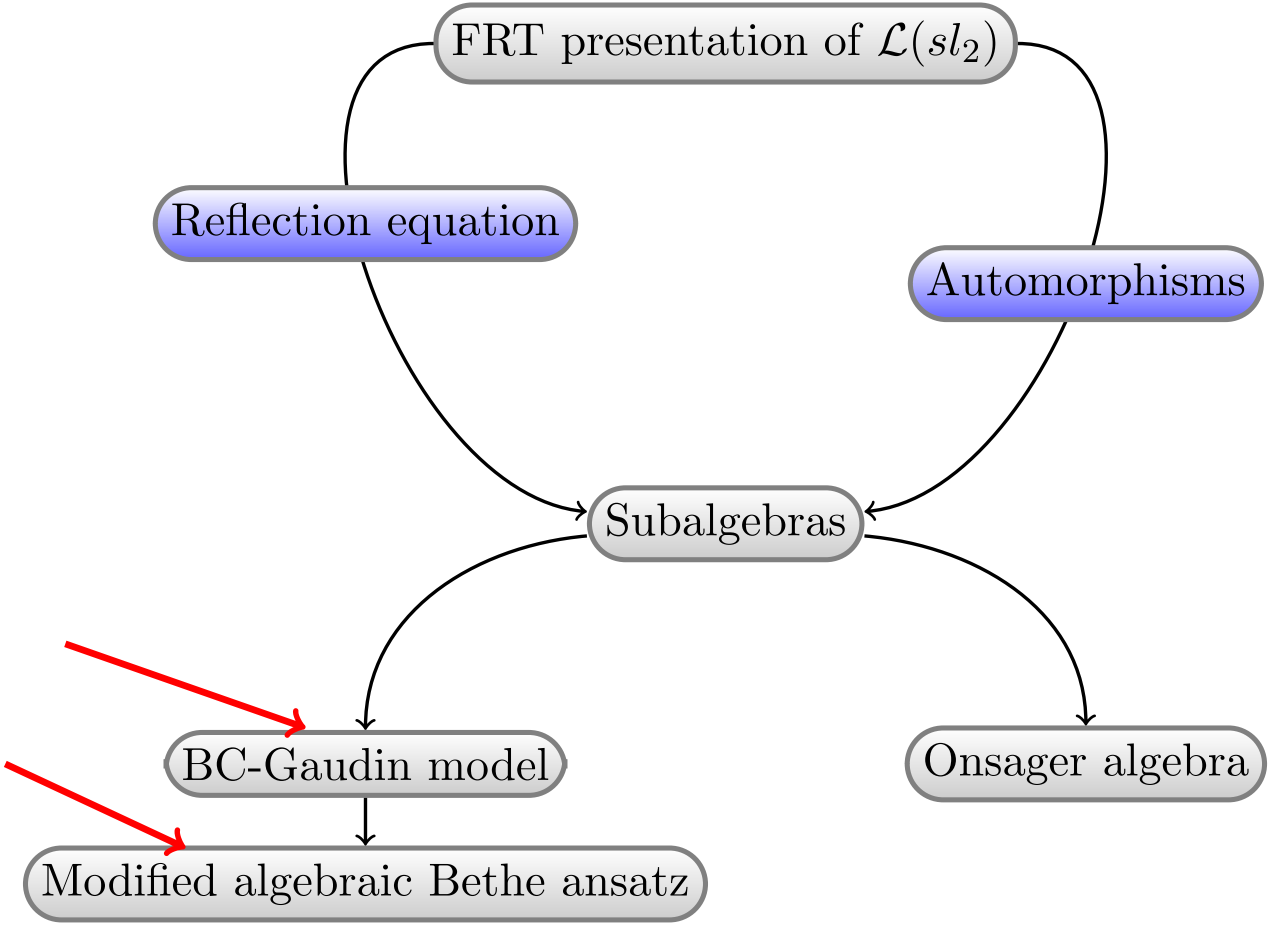
$$k^{(1)}(x) = \begin{pmatrix} \tau(x) & 0 \\ 0 & x \end{pmatrix}$$

Solution for $\mathcal{L}(sl_2)$ of the 3-reflection equation

$$\tau(x) = \frac{ax+b}{cx+d}\, , \quad g^{(1)}(x) = \frac{(a+d)^2 x}{(ax+b)(cx+d)} \quad , \qquad g^{(2)}(x) = ..$$

Constraint $a^2+bc+ad+d^2=0$

$$k^{(1)}(x) = \begin{pmatrix} (cx-a)(dx-b) & 0 \\ 0 & (a+d)(cx+d) \end{pmatrix} \quad , \qquad k^{(2)}(x) = ..$$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BC-Gaudin model

Goal

Compute the eigenvalues and the eigenvectors of the XXZ Gaudin model based on the generic solutions of the reflection equation

BC-Gaudin model

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Compute the eigenvalues and the eigenvectors of the XXZ Gaudin model based on the generic solutions of the reflection equation

$$B_0(x) = \widetilde{r}_{01}(x, v_1) + \widetilde{r}_{02}(x, v_2) + \cdots + \widetilde{r}_{0L}(x, v_L)$$

satisfies (re) where

$$\widetilde{r}_{12}(x,y) = \begin{pmatrix} -\frac{1}{2}\omega(x,y) & 0 & b(x) & 0 \\ 0 & \frac{1}{2}\omega(x,y) & f(y,x) & -b(x) \\ c(x) & -f(1/y,1/x) & \frac{1}{2}\omega(x,y) & 0 \\ 0 & -c(x) & 0 & -\frac{1}{2}\omega(x,y) \end{pmatrix}$$

BC-Gaudin model

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Compute the eigenvalues and the eigenvectors of the XXZ Gaudin model based on the generic solutions of the reflection equation

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Diagonalization of $t(x) = trB(x)^2$?

BC-Gaudin model

Usual algebraic Bethe ansatz

$$B_0(x) = \widetilde{r}_{01}(x, v_1) + \cdots + \widetilde{r}_{0L}(x, v_L) =: \begin{pmatrix} \mathcal{A}(x) & \mathcal{B}(x) \\ \mathcal{C}(x) & \mathcal{D}(x) \end{pmatrix}$$

The usual ansatz consists in

$$V(\{u_1, \ldots, u_m\}) = \mathcal{B}(u_1) \ldots \mathcal{B}(u_m)\ \Omega$$

for Ω a particular vector, called pseudo-vacuum.

BC-Gaudin model

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Problems

- $[\ \mathcal{B}(x)\ ,\ \mathcal{B}(y)\ ]\ \neq\ 0$

BC-Gaudin model

Usual algebraic Bethe ansatz

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$$V(\{u_1, \ldots, u_m\}) = \mathcal{B}(u_1) \ldots \mathcal{B}(u_m)\ \Omega$$

for Ω a particular vector, called pseudo-vacuum.

Problems

- $[\ \mathcal{B}(x)\ ,\ \mathcal{B}(y)\ ]\ \neq\ 0$
- There does not exist simple Ω such that

$$t(x)\ \Omega = \Lambda(x)\ \Omega \qquad \text{and} \qquad \mathcal{C}(x)\ \Omega = 0$$

BC-Gaudin model

Problem 1

$$[\ \mathcal{B}(x)\ ,\ \mathcal{B}(y)\ ]\ \neq\ 0$$

BC-Gaudin model

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$[\ \mathcal{B}(x)\ ,\ \mathcal{B}(y)\ ]\ \neq\ 0$

Solution

We introduce

$$\mathcal{B}(x,n) \quad = \quad \mathcal{B}(x) - (2n-1)b(x)$$

which satisfies

$$\mathcal{B}(x,n)\ \mathcal{B}(y,n+1) \quad = \quad \mathcal{B}(y,n)\ \mathcal{B}(x,n+1)$$

BC-Gaudin model

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Bethe vectors

The Bethe vectors are

$$\mathbb{B}(\{u_1,\dots,u_m\})\Omega = \mathcal{B}(u_1,1)\mathcal{B}(u_1,2)\dots\mathcal{B}(u_m,m)\Omega$$

BC-Gaudin model

Problem 2

There does not exist simple Ω such that

$$t(x)\ \Omega = \Lambda(x)\ \Omega \qquad \text{and} \qquad \mathcal{C}(x)\ \Omega = 0$$

BC-Gaudin model

Problem 2

There does not exist simple Ω such that

$$t(x)\ \Omega = \Lambda(x)\ \Omega \qquad \text{and} \qquad \mathcal{C}(x)\ \Omega = 0$$

After usual computations, one gets, for any m,

$$t(x)\mathbb{B}(\{u_1,\dots,u_m\})\Omega$$

$$=$$

$$\mathrm{W}(x)\ \mathbb{B}(\{u_1,\dots,u_m\})\ \Omega$$

$$+\sum_{p=1}^{m} \mathrm{UW}^{(p)}(x)\ \mathbb{B}(\{u_1,\dots,x,\dots,u_m\})\ \Omega$$

$$+\frac{L-2m-1}{2}\ c(x)\ \mathbb{B}(\{u_1,\dots,u_m\})\ \mathcal{B}(x,m+1)\ \Omega$$

BC-Gaudin model

$$\frac{L-2m-1}{2}\, c(x)\ \mathbb{B}(\{u_1,\ldots,u_m\})\ \mathcal{B}(x,m+1)\ \Omega \ = ?$$

BC-Gaudin model

$$\frac{L-2m-1}{2}\, c(x)\ \mathbb{B}(\{u_1,\dots,u_m\})\ \mathcal{B}(x,m+1)\ \Omega\ = ?$$

- $c(x) = 0$. Triangular boundary

BC-Gaudin model

$$\frac{L-2m-1}{2}\, c(x)\, \mathbb{B}(\{u_1,\dots,u_m\})\, \mathcal{B}(x,m+1)\, \Omega \ = ?$$

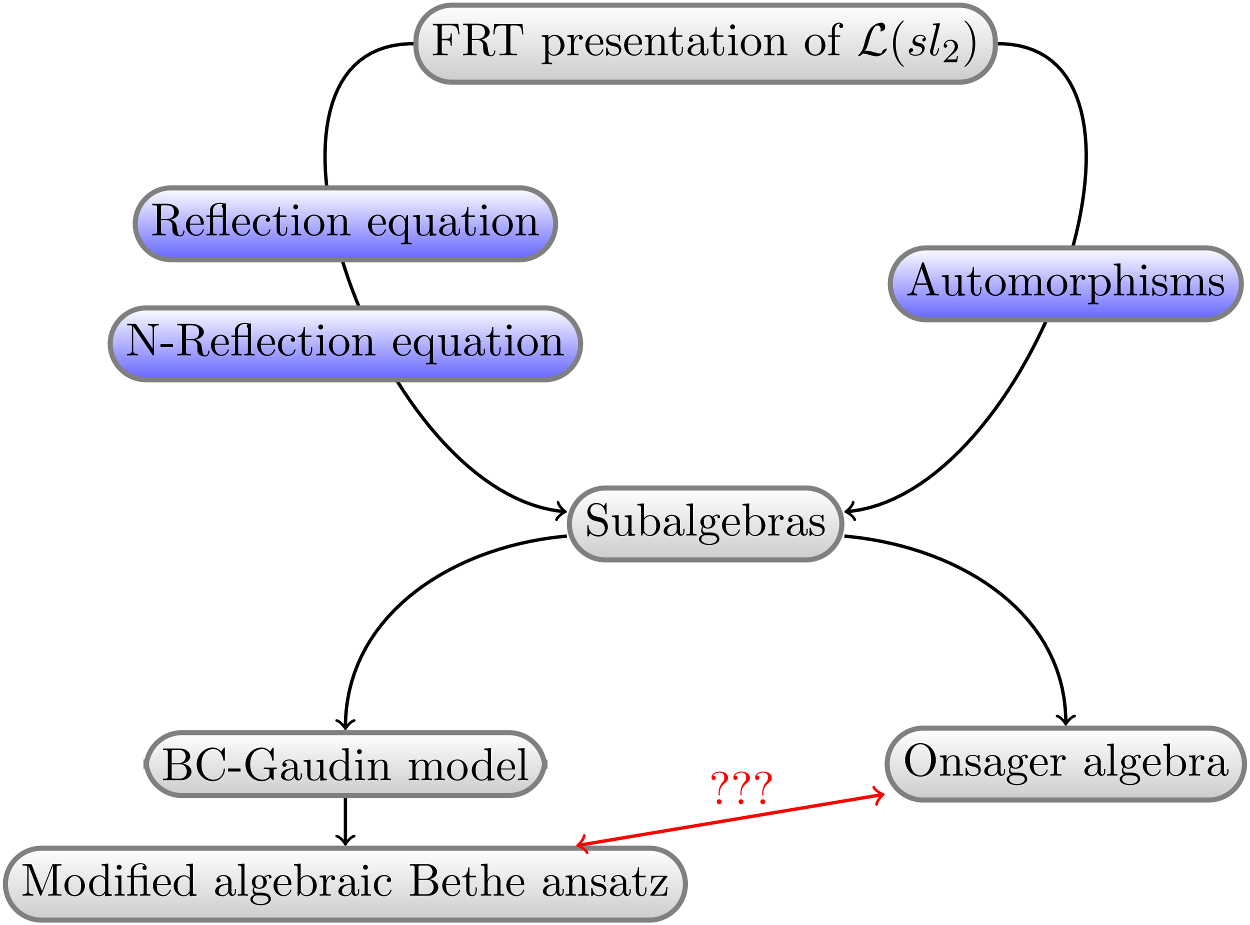
- $c(x) = 0$. Triangular boundary
- $m = \frac{L-1}{2}$. Valid only for L odd.

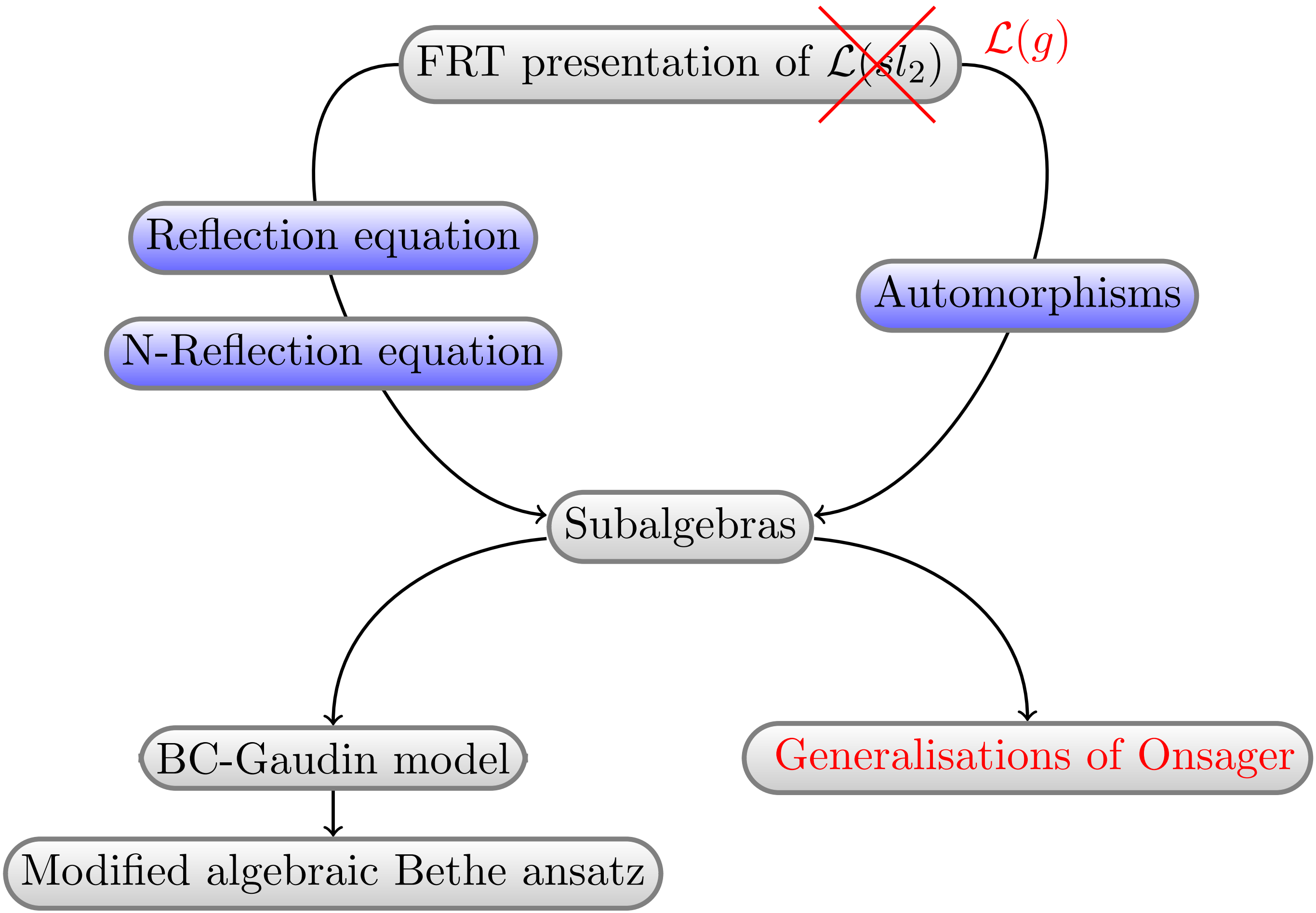
BC-Gaudin model

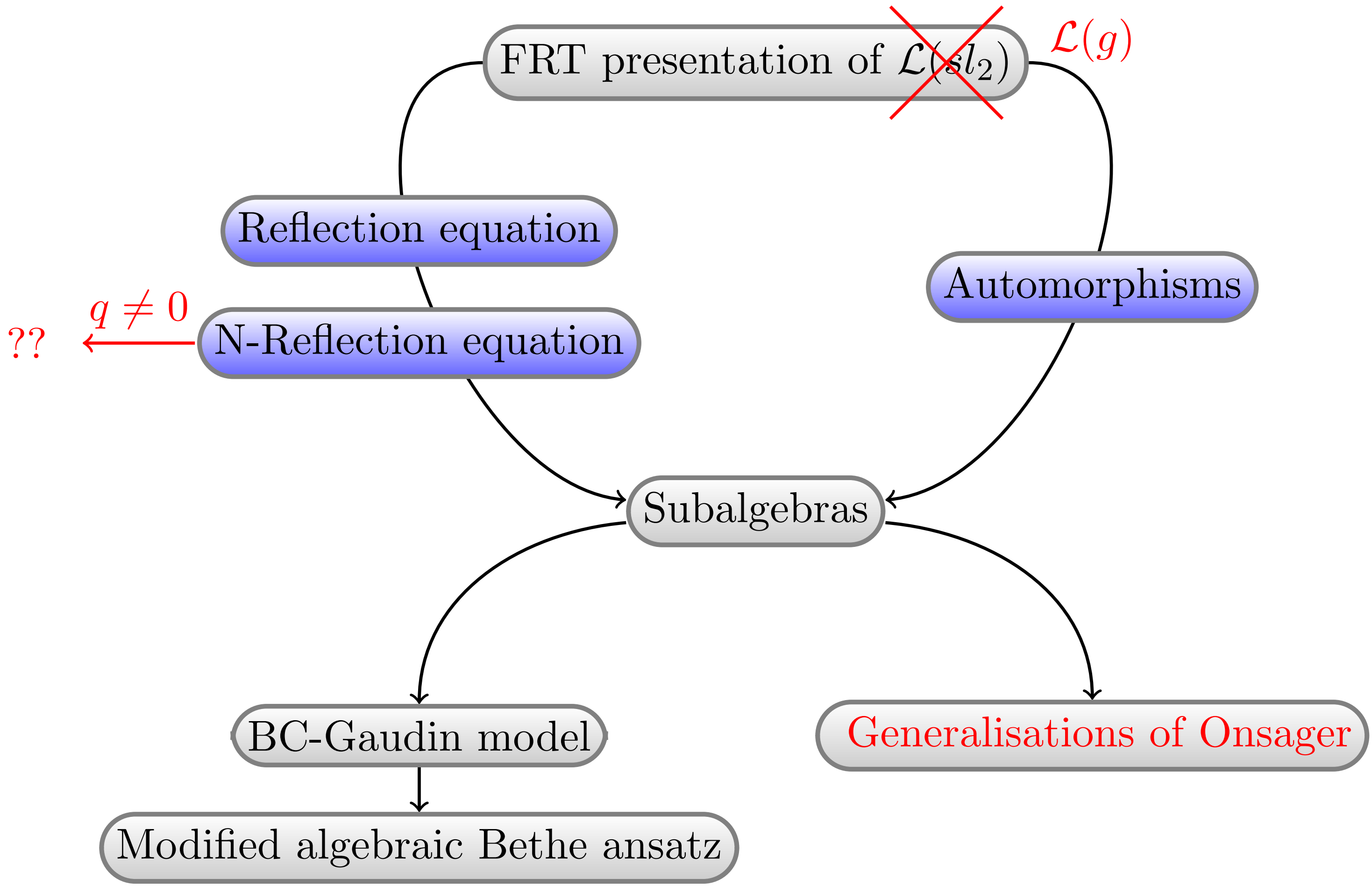
$$\frac{L-2m-1}{2}\, c(x)\ \mathbb{B}(\{u_1,\dots,u_m\})\ \mathcal{B}(x,m+1)\ \Omega\ = ?$$

- $c(x)=0$. Triangular boundary
- $m=\frac{L-1}{2}$. Valid only for L odd.
- Modified Algebraic Bethe Ansatz

$$\mathbb{B}(\{u_1,\dots,u_L\})\ \mathcal{B}(x,L+1)\ \Omega$$
$$=$$
$$\overline{\mathrm{W}}(x)\ \mathbb{B}(\{u_1,\dots,u_L\})\ \Omega$$
$$+\sum_{p=1}^{L}\overline{\mathrm{UW}}^{(p)}(x)\ \mathbb{B}(\{u_1,\dots,x,\dots,u_L\})\ \Omega$$


FRT presentation of $\mathcal{L}(sl_2)$
Reflection equation
N-Reflection equation
Automorphisms
Subalgebras
BC-Gaudin model
Onsager algebra
???
Modified algebraic Bethe ansatz


FRT presentation of $\mathcal{L}(sl_2)$
$\mathcal{L}(g)$
Reflection equation
N-Reflection equation
Automorphisms
Subalgebras
BC-Gaudin model
Generalisations of Onsager
Modified algebraic Bethe ansatz


FRT presentation of $\mathcal{L}(sl_2)$
$\mathcal{L}(g)$
Reflection equation
$q \neq 0$
??
N-Reflection equation
Automorphisms
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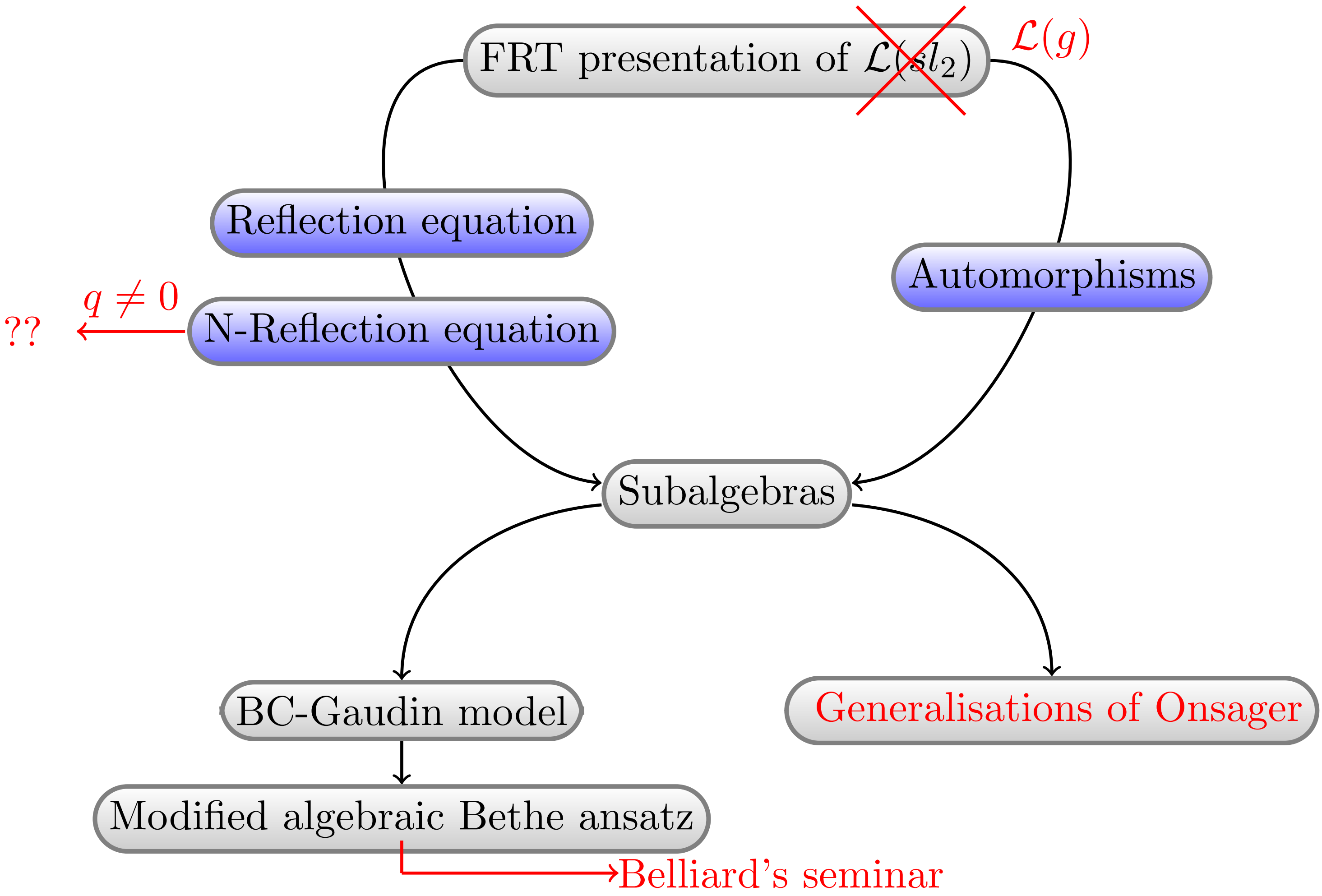
BC-Gaudin model
Generalisations of Onsager
Modified algebraic Bethe ansatz



FRT presentation of $\mathcal{L}(sl_2)$
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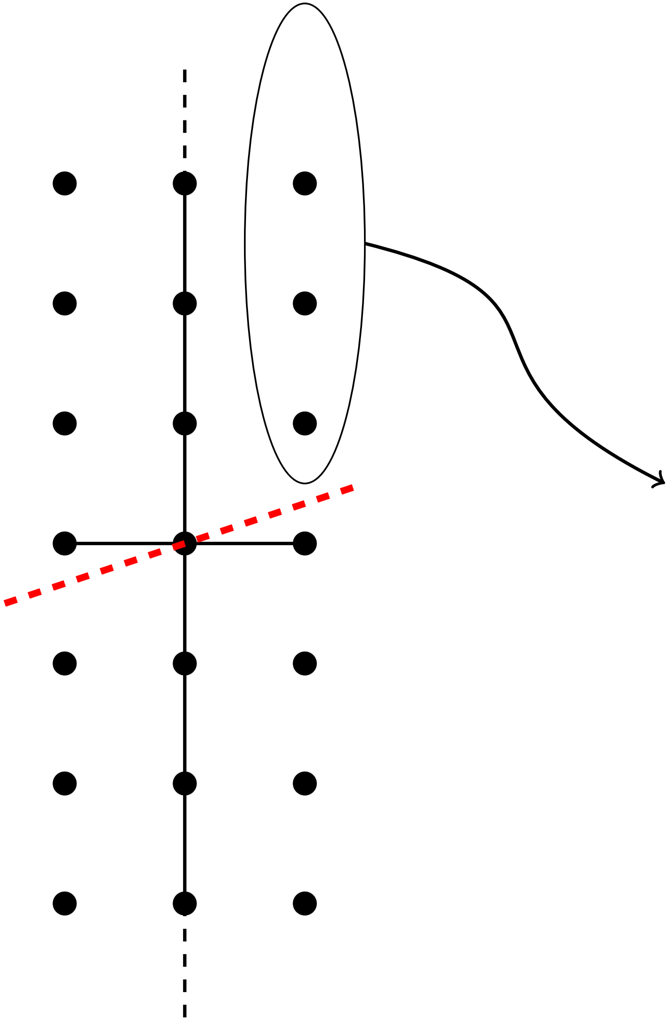
Belliard's seminar



THANK YOU