

# On a conjecture about regularity and $\ell$ -abelian complexity

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A natural generalization of automatic sequences over an infinite alphabet is given by the notion of  $k$ -regular sequences, introduced by Allouche and Shallit in 1992. The  $k$ -regularity of a sequence provides us with structural information about how the different terms are related to each other. We show that a sequence satisfying a certain symmetry property is 2-regular. We apply this theorem to develop a general approach for studying the  $\ell$ -abelian complexity of 2-automatic sequences. In particular, we prove that the period-doubling word and the Thue–Morse word have 2-abelian complexity sequences that are 2-regular. The computation and arguments leading to these results fit into a quite general scheme that can be used to obtain additional regularity results. This supports the conjecture that the  $\ell$ -abelian complexity of a  $k$ -automatic sequence is a  $k$ -regular sequence.

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