

# Dyck paths and Diophantine equations

José Manuel RODRIGUEZ CABALLERO\*

[josephcmac@gmail.com](mailto:josephcmac@gmail.com)

---

I will start with classical results due to Jacobi ( $x^2 + y^2 = n$ ), Dirichlet ( $x^2 + 2y^2 = n$ ) and Lorenz ( $x^2 + 3y^2 = n$ ) which give the number of solutions of the corresponding Diophantine equations in terms of divisor functions. I will present a recent progress due to C. Kassel and C. Reutenauer in 2015 which allows to express the above-mentioned results due to Jacobi and Dirichlet as special values of a sequence of polynomials  $P_n(q)$  related to the zeta function of the Hilbert scheme of  $n$  points on a two-dimensional torus. Finally, using Dyck paths, I will make a contribution by expressing the above-mentioned result of Lorenz as a special value of a polynomial which is related to Kassel-Reutenauer polynomials  $P_n(q)$ .

---

\*LaCIM, UQÀM, CP 8888, succ Centre-ville, Montréal, QC H2 X3Y7