

# From automatic semigroups to automaton semigroups

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We develop an effective and natural approach to interpret any semigroup admitting a special language of greedy normal forms as an automaton semigroup, namely the semigroup generated by a Mealy automaton encoding the behaviour of such a language of greedy normal forms under one-sided multiplication. The framework embraces many of the well-known classes of (automatic) semigroups: free semigroups, free commutative semigroups, trace or divisibility monoids, braid or Artin–Tits or Krammer or Garside monoids, Baumslag–Solitar semigroups, etc. Like plactic monoids or Chinese monoids, some neither left- nor right-cancellative automatic semigroups are also investigated, as well as some residually finite variations of the bicyclic monoid. It provides what appears to be the first known connection from a class of automatic semigroups to a class of automaton semigroups. It is worthwhile noting that, in all these cases, “being an automatic semigroup” and “being an automaton semigroup” become dual properties in a very automata-theoretical sense. Quadratic rewriting systems and associated tilings appear as the cornerstone of our construction.