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Nonlinear Mahler equations and the model theory of difference fields

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Fix two integers k_1 and k_2; two polynomials R_1, R_2 in two variables, over $\mathbb{C}$; and two functions or formal power series f_1, f_2, over $\mathbb{C}$, that satisfy Mahler functional equations

$$f_i(t^{k_i}) = R_i(t, f_i(t)) \text{ for } i = 1, 2.$$

What algebraic relations can hold between f_1 and f_2, over $\mathbb{C}(t)$?

Recent work on k-Mahler functions, with applications to automatic sequences, mostly concerns linear equations in $f(t), f(t^k), f(t^{k^2}), \ldots$. Our equations more closely resemble the ones Mahler originally studied, and we focus instead on the non-linear case, where at least one R_i has degree at least 2 in the second variable.

Our question can be reduced to something involving only one Mahler operator, and then the model theory of difference fields combined with some direct computations show that no algebraic relations can hold between f_1 and f_2, except for very special cases like $k_1 = k_2$ or $R_i(x, y)$ doesn't depend on one of the variables.

This is joint work with Khoa Nguyen and Thomas Scanlon.

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