Chaire Aisenstadt
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Lectures on Automata in Number Theory

Boris ADAMCZEWSKI*

Boris.Adamczewski@math.cnrs.fr

Finite automata form a class of very basic Turing machines. In number theory, they can be used to define in a natural way sequences and sets which are said to be "automatic". One of the main interest of these automatic structures is that they enjoy some strong regularity without being trivial at all. They can be thus though of as lying somewhere between order and chaos, though in many aspects they appear as essentially regular. This special feature of automatic structures leads to various applications of automata theory to number theory.

As part of my Aisenstadt chair, I will give a series of lectures describing some links between these automatic structures and some classical number theoretical problems. Such problems include the representation of integers and real numbers in an integer base, Diophantine equations and decidability, the study of arithmetic differential equations, transcendence and algebraic independence.

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Representing natural numbers in base 2... and 3

In this public lecture, I will discuss a theorem of Alan Cobham that comes back to the late 1960s. This classical result formalizes in terms of finite automata the following naive thought: while it can be readily decided from its binary expansion whether a natural number is a power of 2, it is somewhat harder to derive this information from its ternary expansion. One major interest here is that the pioneering ideas of Cobham has led to a large variety of works including various topics (model theory, tilings, fractals, number theory, difference equations and analysis). I will present the theorem and discuss one generalization which is at the heart of recent works of Bell, Faverjon, Schäfke, Singer, and the speaker.

*Institut Camille Jordan, ICJ, CNRS & Université de Lyon, 43 boulevard du 11 novembre 1918, F-69622 Villeurbanne Cedex, FRANCE

Algebraic power series and Diophantine equations

The Skolem-Mahler-Lech theorem is a classical result concerning the vanishing of linear recurrences over fields of characteristic zero. In this talk, I will discuss some results obtained by Bell, Derksen, Masser, and the speaker which are inspired by analogous number theoretical questions over fields of positive characteristic. I will especially focus on some decision problems related to these questions.

Diagonals of rational functions

A very rich interplay between arithmetic, geometry, transcendence and combinatorics arises in the study of homogeneous linear differential equations and especially of those that "come from geometry" and the related study of Siegel G-functions. A remarkable result is that, by adding variables, we can see many transcendental G-functions as arising in a natural way from much more elementary function, namely rational functions. This process, called diagonalization, can be thought of as a formal integration.

I will discuss some arithmetical problems that are related to diagonals of rational functions, their reduction mod p and their link with automata theory. This corresponds to some joint works with Bell and Delaygue.

Mahler's method and automatic numbers

A Mahler function is a solution, analytic in some neighborhood of the origin, of a linear difference equation associated with the Mahler operator $z \mapsto z^q$, where $q \geq 2$ is an integer. Understanding the nature of such functions at algebraic points of the complex open unit disc is an old number theoretical problem dating back to the pioneering works of Mahler in the late 1920s. In these talks, I will first explain why it can be reasonably considered as totally solved now, after works of Ku. Nishioka, Philippon, Faverjon and the speaker. I will also describe the consequences of Mahler's method for the study of the decimal expansion of algebraic irrational numbers. I will finally discuss some work in progress with Faverjon, as well as perspective, regarding Mahler's method in higher dimension and its application to automata theory and automatic numbers.