

# Invariant Derivations and Differential Forms for Reflection Groups

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Classical invariant theory of a complex reflection group  $W$  highlights three beautiful structures:

- the  $W$ -invariant polynomials constitute a polynomial algebra, over which
- the  $W$ -invariant differential forms with polynomial coefficients constitute an exterior algebra, and
- the relative invariants of any  $W$ -representation constitute a free module.

When  $W$  is a duality (or well-generated) group, we give an explicit description of the isotypic component within the differential forms of the irreducible reflection representation. This resolves a combinatorial conjecture motivated by  $W$ -Catalan combinatorics, and also relates to a certain thread of results in Lie theory.

We establish this result in a case-free fashion, by examining the space of  $W$ -invariant differential derivations; these are derivations whose coefficients are not just polynomials, but differential forms with polynomial coefficients. . . When  $W$  is a duality group, we show that the space of invariant differential derivations is free as a module over the exterior subalgebra of  $W$ -invariant forms generated by all but the top-degree exterior generator. (The basic invariant of highest degree is omitted.)

*This is joint work with Anne Shepler.*

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