

Invariant Derivations and Differential Forms for Reflection Groups

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Classical invariant theory of a complex reflection group W highlights three beautiful structures:

- the W -invariant polynomials constitute a polynomial algebra, over which
- the W -invariant differential forms with polynomial coefficients constitute an exterior algebra, and
- the relative invariants of any W -representation constitute a free module.

When W is a duality (or well-generated) group, we give an explicit description of the isotypic component within the differential forms of the irreducible reflection representation. This resolves a combinatorial conjecture motivated by W -Catalan combinatorics, and also relates to a certain thread of results in Lie theory.

We establish this result in a case-free fashion, by examining the space of W -invariant differential derivations; these are derivations whose coefficients are not just polynomials, but differential forms with polynomial coefficients. . When W is a duality group, we show that the space of invariant differential derivations is free as a module over the exterior subalgebra of W -invariant forms generated by all but the top-degree exterior generator. (The basic invariant of highest degree is omitted.)

This is joint work with Anne Shepler.

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