

# Lattice congruences of the weak order

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These lectures are an introduction to lattice-theoretic concepts in the study of Coxeter groups. Our first order of business is to present the lattice-theoretic “facts of life” with an emphasis on the ideas most relevant to studying the weak order on a finite Coxeter group. We then apply these ideas to the weak order, motivating our study with examples of lattice congruences and lattice quotients of the weak order. (Knowing the facts of life, we are able to compute specific examples in low rank.) We present a combinatorial model (noncrossing arc diagrams) for lattice congruences/quotients of the weak order on permutations and spend some time on the Cambrian congruences/Cambrian lattices. Time allowing, we conclude the lectures by placing the lattice theory in the geometric setting of hyperplane arrangements and “shards.”

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