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Lattice homomorphisms between weak orders and between Cambrian lattices

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Given a finite Coxeter group W, the automorphisms of the weak order on W are exactly the diagram automorphisms (the symmetries of the Coxeter diagram of W). In this talk, we consider a larger class of maps: surjective lattice homomorphisms between the weak orders on different finite Coxeter groups. Just as for automorphisms, everything is essentially determined by acting on Coxeter diagrams. Specifically, a surjective homomorphism exists from W to W' if and only if the diagram for W' can be obtained from the diagram for W by deleting vertices and/or decreasing edge labels (and thus possibly erasing edges). Once such an operation on diagrams is decided, the homomorphism is completely determined by its restriction to rank-2 standard parabolic subgroups. Furthermore, W' is a very simply-described lattice quotient of W. (The analogy is that we would call R' a simply-described quotient of a ring R if $R' = R/I$ for I generated by a few low-degree polynomials.)

Our classification of surjective lattice homomorphisms between finite Coxeter groups leads easily to an even nicer classification of surjective lattice homomorphisms between Cambrian lattices in terms of "oriented diagram homomorphisms." If there is time, I'll mention some applications to cluster algebras, namely refinement relations among **g**-vector fans and surprising ring homomorphisms between cluster algebras.

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