

From Generalized Permutahedra to Grothendieck Polynomials via Flow Polytopes

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We prove that the Grothendieck polynomial $\mathfrak{G}_{1\pi'}(\mathbf{x})$, for permutations $1\pi'$, where π' is dominant, is a weighted lattice point enumerator of its Newton polytope, with all weights nonzero. We also show that the Newton polytopes of the homogeneous components of $\mathfrak{G}_{1\pi'}(\mathbf{x})$ are generalized permutahedra. Moreover, the Schubert polynomial $\mathfrak{S}_{1\pi'}(\mathbf{x})$, for permutations $1\pi'$, where π' is dominant, equals the lattice point enumerator of a generalized permutahedron. These results imply recent conjectures of Monical, Tokcan and Yong regarding the supports of Schubert and Grothendieck polynomials for permutations $1\pi'$, where π' is dominant. We connect Grothendieck polynomials and generalized permutahedra via a family of dissections of flow polytopes obtained from the subdivision algebra. We naturally label each simplex in a dissection by a sequence, called a left-degree sequence, and show that the left-degree sequences arising from simplices of a fixed dimension in our dissections of flow polytopes are exactly the integer points of generalized permutahedra. We also connect left-degree sequences and Grothendieck polynomials, thereby revealing the beautiful relation between generalized permutahedra and Grothendieck polynomials.

The talk is based on joint papers with Laura Escobar and Avery St. Dizier.

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