"Théories de jauge, monopoles, espaces de modules et systèmes intégrables"
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Symplectic Geometry of the Moduli Space of Projective Structures on Riemann Surfaces

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The bundle of quadratic differentials with simple zeroes over the moduli space of Riemann surfaces can be endowed with a natural symplectic structure, which we call "homological symplectic structure" in terms of explicit Darboux coordinates. On the other hand, the *vector* bundle of quadratic differentials is a model of the cotangent bundle of the same moduli space, and hence it carries the canonical symplectic structure. The two structures coincide in the common domain, and hence we provide Darboux coordinates for the canonical structure as well.

In addition, the affine bundle of projective connections is modelled on the bundle of quadratic differentials. By choosing a holomorphically varying base projective connection, the projective connections and the space of quadratic differentials can be naturally (but not canonically) identified. This allows to induce symplectic structures also on the moduli space of projective connections.

Moreover, a projective connection defines a monodromy map and hence a point in the character variety, i.e., homomorphism of the fundamental group of a Riemann surface into unimodular two by two matrices modulo conjugations.

Goldman ('86) introduced a Poisson bracket on the character variety which, in this case, is symplectic.

Then the result is that the push forward of the homological symplectic structure to the character variety (by an appropriate choice of base projective connection) coincides with the Goldman bracket.

I hope to define all the necessary objects and be as elementary as possible.

This is joint work with Chaya Norton (*Concordia*) *and Dmitry Korotkin* (*Concordia*).

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