Distress and Default Contagion in Financial Networks

Luitgard A. M. Veraart

London School of Economics and Political Science

September 2017

Workshop: Measurement and Control of Systemic Risk; CRM Montreal, September 2017

Preprint available at SSRN: https://ssrn.com/abstract=3029702

Systemic risk in financial networks

- Aim: Analyse stability of financial networks through stress tests.
- Financial institutions connected through different channels: interbank lending, common asset holdings etc.
- Describe connections by network model.
- Potential for domino effects (contagion) of losses spreading through the network.
- How to model contagion?

Modelling solvency contagion

- What triggers contagion?
  - Classical models (default contagion): Only bankruptcy triggers contagion, Eisenberg & Noe (2001); Furfine (2003); Gai et al. (2011); Amini et al. (2016); Rogers & V. (2013); Kusnetsov & V. (2016).
  - New models (distress contagion): Contagion can start prior to default event (marking to markets), Battiston et al. (2012); Barucca et al. (2016); Bardoscia et al. (2017); Glasserman & Young (2015), V. (2017).
    "During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults", Basel Committee on Banking Supervision (2011).
- Spread of contagion? Are losses: passed on, contained, amplified?
- Relationship between trigger and spread of contagion?

The financial market

- Consider interbank market as network:
  - Nodes consist of N banks.
  - Directed weighted edges $L_{ij} \geq 0$ represent nominal interbank liability of bank i to bank j.
- Each node i has stylised balance sheet:

Assets		Liabilities	
shocked external assets	$A_i^{(e)} - x_i$	external liabilities	$L_i^{(e)}$
interbank assets	$\sum_{j=1}^{N} L_{ji}$	interbank liabilities	$\sum_{j=1}^{N} L_{ij}$
		shocked net worth	$w_i - x_i$

$$\text{net worth: } w_i = \underbrace{A_i^{\mathrm{e}} + \sum_{j=1}^{N} L_{ji}}_{=:\bar{A}_i} - \underbrace{\left(L_i^{\mathrm{e}} + \sum_{j=1}^{N} L_{ij} \right)}_{=:\bar{L}_i},$$

Clearing and network revaluation

- Who pays whom and how much?
- $$\text{Amount paid by } i = \begin{cases} \bar{L}_i, & \text{if possible,} \\ \text{determined by clearing,} & \text{else.} \end{cases}$$
- Clearing:
  - Clearing mechanism determines payments between banks (Eisenberg & Noe, 2001).
  - Can also be interpreted as a revaluation of a network faced by a shock, Glasserman & Young (2015).
- Relative liabilities matrix $\Pi \in \mathbb{R}^{n \times n}$,

$$\Pi_{ij} := \begin{cases} L_{ij}/\bar{L}_i, & \text{if } \bar{L}_i > 0, \\ 0, & \text{if } \bar{L}_i = 0. \end{cases}$$

The clearing mechanism (Eisenberg & Noe, 2001); (Rogers & V., 2013)

- Limited liabilities: Nodes never pay more than available cash flow.
- Proportionality: Defaulting bank pays all claimant banks in proportion to size of their nominal claims on the assets of defaulting bank.
- Exogenous recovery rates: (Rogers & V. 2013) $\alpha, \beta \in [0,1]$ (determine default costs).
- A clearing vector for the financial system with shock realisation x is a vector $L(x) \in [0, \bar{L}]$ such that $L(x) = \Psi^{\mathrm{RV}}(L(x))$, where

$$\Psi^{\mathrm{RV}}(L(x))_i := \begin{cases} \bar{L}_i, & \text{if} \quad \bar{L}_i \leq A_i^{(e)} - x_i + \sum_{j=1}^n L_j(x)\Pi_{ji}, \\ \alpha(a_i^{(e)} - x_i) + \beta \sum_{j=1}^n L_j(x)\Pi_{ji}, & \text{else}. \end{cases}$$

Clearing and and re-evaluated equity

- Idea: Rewrite fixed point problem in clearing payments as a fixed point problem in re-evaluated equity.
- Related work: Barucca et al. (2016), Hurd (2016).
- *Admissible valuation function*: $\mathbb{V} : \mathbb{R} \to [0, 1]$ nondecreasing, right-continuous.
- *Equity valuation function*: $\Phi = \Phi(\cdot; \mathbb{V}) : \mathcal{E}(x) \to \mathcal{E}(x)$, $\forall i \in \mathcal{N}$

$$\Phi_i(E) = \Phi_i(E; \mathbb{V}) = \underbrace{A_i^{\mathrm{e}} - x_i + \sum_{j \in \mathcal{M}} L_{ji} \mathbb{V}\left(\frac{E_j + \bar{L}_j}{\bar{L}_j}\right)}_{re-evaluated assets} - \underbrace{\bar{L}_i}_{liabilities}, \quad (1)$$

$(L, L^{\mathrm{e}}, A^{\mathrm{e}})$ financial system; shock vector $x \in [0, A^{(\mathrm{e})}]$; $\mathbb{V}$ admissible valuation function; $\mathcal{E}(x) = [-\bar{L}, w - x]$; $\mathcal{M} := \{j \in \mathcal{N} \mid \bar{L}_j > 0\}$.

- *Re-evaluated equity*: $E \in \mathcal{E}(x)$ with $E = \Phi(E)$.

The Rogers & V. (2013) model as equity revaluation

Lemma

Let (L, L^e, A^e) *be a financial system with shock vector* $x \in [0, A^{(e)}]$ *and let* $\mathcal{M} := \{j \in \mathcal{N} \mid \bar{L}_j > 0\}$.
Then, the function $\mathbb{V}^{RV} : \mathbb{R} \to [0, 1]$ *given by*

$$\mathbb{V}^{RV}(y) = \begin{cases} 1 & \text{if } y \geq 1, \\ \beta y^+ & \text{if } y < 1, \end{cases} \tag{2}$$

is an admissible valuation function, and

$$\Phi_i^{RV}(E) = A_i^e - x_i + \sum_{j \in \mathcal{M}} L_{ji} \mathbb{V}^{RV}\left(\frac{E_j + \bar{L}_j}{\bar{L}_j}\right) - \bar{L}_i, \tag{3}$$

is an equity valuation function.

Theorem: Relationship clearing payment and equity revaluation in Rogers & V. (2013) model I

Let $(L, L^{\mathrm{e}}, A^{\mathrm{e}})$ be a financial system with shock vector $x \in [0, A^{(\mathrm{e})}]$ and $\mathcal{M} = \{i \in \mathcal{N} \mid \bar{L}_i > 0\}$. Let $\alpha = \beta \in [0, 1]$.

1. Let $L^*(x)$ be a fixed point of Ψ^{RV}. Then, E^* given by

$$E_i^* := A_i^{\mathrm{e}} - x_i + \sum_{j \in \mathcal{M}} L_{ji} \frac{L_j^*(x)}{\bar{L}_j} - \bar{L}_i, \quad \forall i \in \mathcal{N} \tag{4}$$

   is a fixed point of Φ^{RV}.

2. Let $L^*(x)$ be the greatest fixed point of Ψ^{RV}. Then E^* defined in (4) is the greatest fixed point of Φ^{RV}.

Theorem: Relationship clearing payment and equity revaluation in Rogers & V. (2013) model II

3. Let E^* be a fixed point of Φ^{RV}. Then $L^*(x)$ given by

$$L^*(x)_i = \mathbb{V}^{\mathrm{RV}}\left(\frac{E_i^* + \bar{L}_i}{\bar{L}_i}\right)\bar{L}_i \quad \forall i \in \mathcal{N} \tag{5}$$

   is a fixed point of Ψ^{RV}.

4. Let E^* be the greatest fixed point of Φ^{RV}. Then $L^*(x)$ given by (5) is the greatest fixed point of Ψ^{RV}.

Accounting for distress contagion

- Valuation function $\mathbb{V} : \mathbb{R} \to [0, 1]$, where in classical models (Eisenberg & Noe (2001); Rogers & V. (2013)) $\mathbb{V}(y) = 1$ for $y \geq 1 \Leftrightarrow \text{total asset value} \geq \bar{L}$.

- What if marking to markets reduces asset value prior to $y = 1$?

- Assume existence of capital cushion parameter $k \in [0, \infty)$ as in Glasserman & Young (2015). Decline starts when assets fall below higher value $(1 + k)\bar{L}$.

- Consider function $\mathbb{V}^{\text{Distress}} : \mathbb{R} \to [0, 1]$ with

$$\mathbb{V}^{\text{Distress}}(y) = \mathbb{I}_{\{y \geq 1+k\}} + \mathbb{I}_{\{y < 1+k\}} r(y), \tag{6}$$

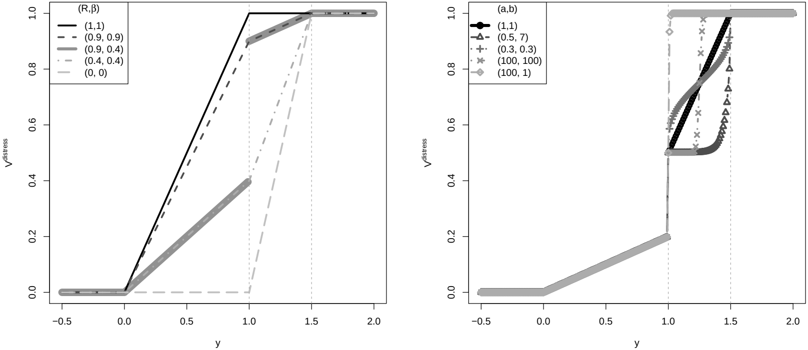
where $r : \mathbb{R} \to [0, 1]$ non-decreasing, right-continuous.

A model for default and distress contagion

- Default contagion branch: Use Rogers & V. (2013) model with $\alpha = \beta \in [0, 1]$.
- Distress contagion branch: Reduced form approach.
- $$r(y) = \begin{cases} 1 - (1 - R)F\left(\frac{1+k-y}{k}; a, b\right), & \text{if } 1 \leq y < 1 + k, \\ \beta y^+, & \text{if } y < 1, \end{cases}$$

  where $y^+ = \max\{y, 0\}$ and F is the cumulative distribution function (cdf) of the Beta distribution with parameters $a > 0, b > 0$.

Sensitivity of $\mathbb{V}^{Distress}$ in (R, β) and (a, b)



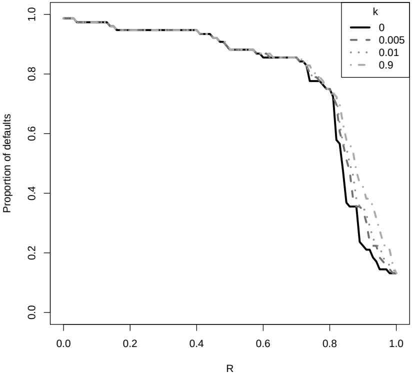
Meaning of model parameters

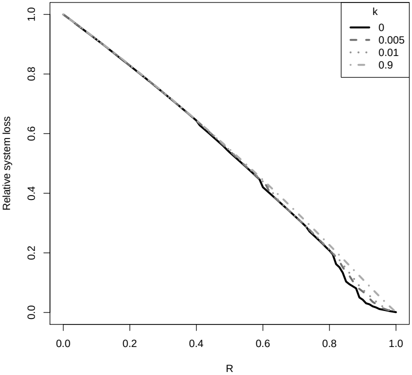
- $k \in [0, \infty)$: parameter models the capital cushion. It determines the start point of contagion.
- $R, \beta \in [0, 1]$ with $\beta \leq R$: parameters modelling the perceived and actual exogenous recovery rates (and determining the perceived and actual proportional default costs).
- $a, b \in (0, \infty)$: parameters modelling decline in asset value due to distress contagion by determining the shape of the cdf of the Beta distribution.
- Distress contagion can only occur if $R < 1$ and $k > 0$.
- Consider Eisenberg & Noe (2001) valuation function (i.e., $R = \beta = 1$) and allow for $k > 0$. No default costs, hence distress contagion cannot occur!

Empirical case study

- Application of valuation function $\mathbb{V}^{\mathrm{Distress}}$ to empirical data.
- Balance sheet data of 76 banks that took part in the European Banking Authority's (EBA) 2011 stress test.
- Individual interbank liabilities L_{ij} not observable.
- Can reconstruct the L_{ij} from the row and column sums of L using Bayesian (MCMC) approach by Gandy & V. (2016, 2017).
- Consider shock $x = 0.03A^{\mathrm{e}}$ which causes negative net worth of 10 banks.
- What happens to remaining 66 banks?

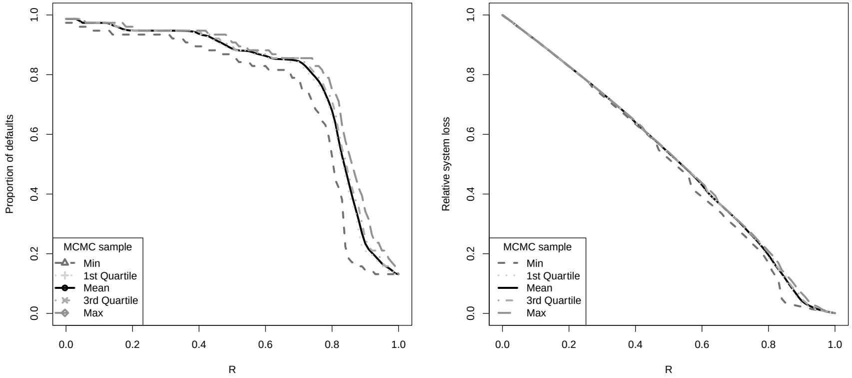
Proportion of defaults and relative system loss based on one network





Proportion of defaults (left) and relative system loss(right) as a function in R for different values of the cushion parameter k using $\beta = R$ and $a = b = 1$ for one reconstructed network.

Quantiles of proportion of defaults and relative system loss based on sample of networks



Minimum, first quartile, mean, third quartile and maximum of the MCMC sample of 10,000 reconstructed networks of the proportion of defaults (left) and relative system loss (right) as a function in R for $\beta = R$ and $a = b = 1$ and $k = 0$.

Conclusion

- New model for distress and default contagion captures wide range of possible contagion mechanisms.
- Only five intuitive model parameters: capital cushion parameter k, the exogenous recovery rates β, R, parameters for shape of distress contagion branch $a, b > 0$.
- Importance of default costs for distress contagion.
- Can be used in stress tests and for sensitivity studies.
- Analytical ordering results and bounds on outcome measures of stress tests for different network revaluation functions.
- Ordering results independent of network structure.

References I

Amini, H., Cont, R. & Minca, A. (2016). Resilience to contagion in financial networks. *Mathematical Finance* **26**, 329–365.

Bardoscia, M., Barucca, P., Codd, A. B. & Hill, J. (2017). The decline of solvency contagion risk. Bank of England Staff Working Paper No. 662.

Barucca, P., Bardoscia, M., Caccioli, F., D'Errico, M., Visentin, G., Battiston, S. & Caldarelli, G. (2016). Network valuation in financial systems. Available at SSRN: https://ssrn.com/abstract=2795583.

Basel Committee on Banking Supervision (2011). Press release. Available at: http://www.bis.org/press/p110601.htm.

References II

Battiston, S., Puliga, M., Kaushik, R., Tasca, P. & Caldarelli, G. (2012). Debtrank: Too central to fail? Financial networks, the FED and systemic risk. *Scientific Reports* **2**.

Eisenberg, L. & Noe, T. H. (2001). Systemic risk in financial systems. *Management Science* **47**, 236–249.

Furfine, C. H. (2003). Interbank exposures: quantifying the risk of contagion. *Journal of money, credit and banking* 111–128.

Gai, P., Haldane, A. & Kapadia, S. (2011). Complexity, concentration and contagion. *Journal of Monetary Economics* **58**, 453–470.

Gandy, A. & Veraart, L. A. M. (2016). A Bayesian methodology for systemic risk assessment in financial networks. *Management Science* Articles in Advance, http://dx.doi.org/10.1287/mnsc.2016.2546.

References III

Gandy, A. & Veraart, L. A. M. (2017). Adjustable network reconstruction with applications to CDS exposures. Available at SSRN: https://ssrn.com/abstract=2895754.

Glasserman, P. & Young, H. P. (2015). How likely is contagion in financial networks? *Journal of Banking & Finance* **50**, 383–399.

Hurd, T. R. (2016). *Contagion! Systemic Risk in Financial Networks*. Springer Briefs in Quantitative Finance, Springer.

Kusnetsov, M. & Veraart, L. A. M. (2017). Interbank clearing in financial networks with multiple maturities. Available at SSRN: https://ssrn.com/abstract=3002637.

Rogers, L. C. G. & Veraart, L. A. M. (2013). Failure and rescue in an interbank network. *Management Science* **59**, 882–898.

References IV

Veraart, L. A. M. (2017). Distress and default contagion in financial networks. Available at SSRN: https://ssrn.com/abstract=3029702.

The Beta distribution

Recall that the probability density function of the Beta distribution with parameters $a > 0$ and $b > 0$ is given by

$$f(x; a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}\mathbb{I}_{\{0 \leq x \leq 1\}}$$

and the corresponding cumulative distribution function is

$$F(x; a, b) = \int_0^x f(y; a, b) dy$$

for $0 \leq x \leq 1$. The corresponding mean is $a/(a+b)$ and the variance is $ab/[(a+b)^2(a+b+1)]$.