Beauty of Discrete Mathematics – Beauté des mathématiques discrètes

Centre de **R**echerches **M**athématiques

Montréal

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Abstracts/Résumés

For full affiliations please see the list of participants.

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Laurent Beaudou

Clermont-Ferand

Absorbing in colours

Given a colouring of the arcs of a digraph, we say that a vertex x *is absorbed* by a vertex y if there is a monochromatic path from x to y. A subset S of V is called an *absorbing set* if any vertex in $V \setminus S$ is absorbed by some vertex in S.

In the early eighties, Sands, Sauer and Woodrow [3] suggested the following problem (also attributed to Erdős in the same paper):

> Problem. [Sands, Sauer and Woodrow [3]] For each n, is there a (least) positive integer $f(n)$ so that every finite tournament whose arcs are coloured with n colours contains an absorbing set S of size $f(n)$?

In 2017, Bousquet Lochet and Thomassé [2] gave a positive answer to the problem of Sands, Sauer and Woodrow.

> Theorem. [Bousquet, Lochet and Thomassé [2]] Function f is well defined and $f(n) = O(\ln(n) \cdot n^{n+2})$.

In this talk, we give an overview of the problem, underline the nice aspects of some of the proofs involved, and provide a lower bound on the value of $f(n)$ [1].

References

[1] L. Beaudou, L. Devroye, and G. Hahn, *A lower bound on the size of an absorbing set in an arc-coloured tournament*, arXiv:1708.08891, 2017.

[2] N. Bousquet, W. Lochet and S. Thomassé, *A proof of the Erdős-Sands-Sauer-Woodrow conjecture*, arXiv:1703.08123v1, 2017.

[3] B. Sands, N. Sauer and R. Woodrow, On monochromatic paths in edge-colored digraphs, *Journal of Combinatorial Theory, Series B*, vol. 33, pp. 271–275, 1982.

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Adrian Bondy

Paris

Mathematicians I have known

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Dwight Duffus

Atlanta

The width of downsets in Boolean lattices

We are motivated by an old problem due to Daykin and Frankl: what is the minimum width of a convex subset of fixed size in a Boolean lattice B(n)? We address this problem for downsets of B(n) (with moderate success) and we fully describe the downsets that maximize width (again, of fixed size in a fixed lattice).

This is joint work with Dave Howard (Colgate University) and Imre Leader (University of Cambridge).

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Przemek Gordinowicz

Łódź

The beauty of the generic poset and its automorphism group

One of the most beautful results in the theory of random graphs is given by Erdős and Rényi (1963) probabilistic construction of countable universal homogeneous graph, called from this reason *the random graph*. The random graph is obtained, with probability 1, as a limit of the $G(n, p)$ model where n tends to ∞ while $p \in (0, 1)$ is fixed. On the other hand it is a Fraïsé limit of the family of all finite graphs.

A natural question is whether Fraïsé limits of another relational structures (eg. generic poset or Henson graphs) can be constructed probabilistically. This question was answered affirmatively by Droste and Kuske [1]. However, it seems that properties of finite structures in their random model were not studied. During the talk we discuss Droste-Kuske approach (applied to posets) and some other model, in which a linear extension to generated poset is given in advance.

In the second part of the talk, based on [2], we show the construction of 2-generated, dense and free subgroup of the authomorphism group of the generic poset.

References

[1] M. Droste, D. Kuske, On random relational structures, *J. Combin. Theory Ser. A* **102** (2003), 241-âĂŞ254.

[2] S. Głąb, P. Gordinowicz, F. Strobin, Dense free subgroups of automorphism groups of homogeneous partially ordered sets, submitted, arxiv:1708.00746.

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Hamed Hatami

Montreal

Polynomial method and graph bootstrap percolation

We introduce a simple method for proving lower bounds for the size of the smallest percolating set in a certain graph bootstrap process. We apply this method to determine the sizes of the smallest percolating sets in multidimensional tori and multidimensional grids (in particular hypercubes). The former answers a question of Morrison and Noel, and the latter provides an alternative and simpler proof for one of their main results. This is based on a joint work with Lianna Hambardzumyan and Yingjie Qian.

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Jan van den Heuvel

London

Improper Colourings inspired by Hadwiger's Conjecture

Hadwiger's Conjecture (1943) asserts that every graph without the complete graph K_t as a minor has a proper vertex-colouring using at most $t-1$ colours. Since the conjecture is stubbornly refusing to be proved, we might look at relaxed versions of it. In this talk we relax the conclusion of the conjecture by considering two types of improper colourings for K_t-minor-free graphs: (1) colourings in which each monochromatic component has small degree, and (2) colourings in which each monochromatic component has small size. In both cases our new results greatly improve the existing results on these colourings. The very simple proofs are based on an elementary decomposition result for graphs without K_t-minor that might be of independent interest. This is joint work with David R. Wood (Monash Univ.)

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Jing Huang

Victoria

Orientation completion problems

For a fixed class C of oriented graphs, the orientation completion problem asks whether a given partially oriented graph P can be completed to an oriented graph in C by orienting the (non-oriented) edges in P. Orientation completion problems commonly generalize several existing problems including recognition of certain graph classes as well as extending representations of certain geometrically representable graphs. We study orientation completion problems for several well-studied classes of oriented graphs, and show how some of the orientation techniques can be employed to solve these problems in polynomial time.

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Jan Hubička

Prague

On the existence of Ramsey expansions

Class $\mathcal{K}$ of finite structures is Ramsey if for every choice of A and B in $\mathcal{K}$ there exists C in $\mathcal{K}$ such that for every coloring of its substructures isomorphic to A with 2 colors there exists an isomorphic copy of B in C where all copies of A are monochromatic. We will discuss sufficient conditions for a class to be Ramsey and show also negative results on non-existence of precompact Ramsey expansion for Hrushovski predimension construction. This is joint work with David Evans, Jaroslav Nešetřil and Matěj Konečný.

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Pierre Ille

Marseille

Fractal graphs

The lexicographic sum of graphs is defined as follows. Let G be a graph. With each vertex v of G associate a graph H_v. The lexicographic sum of the graphs H_v over G is obtained from G by substituting each vertex v of G by H_v. When all the graphs H_v are isomorphic to a same graph H, the lexicographic sum of the graphs H_v over G is called the lexicographic product of H by G, and denoted by $G \wr H$.

We say that a graph G is fractal if there exists a graph Γ, with at least two vertices, such that $G \simeq \Gamma \wr G$. Fractal graphs can be constructed simply. Let Γ be a graph with at least two vertices. Consider the set $V(\Gamma)^\omega$ of the functions from ω to $V(\Gamma)$. The graph Γ^ω is defined on $V(\Gamma)^\omega$ as follows. Given distinct $f, g \in V(\Gamma)^\omega$, fg is an edge of Γ^ω if $f(m)g(m)$ is an edge of Γ, where m is the smallest integer such that $f(m) \neq g(m)$. The graph Γ^ω is fractal because $\Gamma \wr \Gamma^\omega \simeq \Gamma^{1+\omega} \simeq \Gamma^\omega$.

We prove that a fractal graph is isomorphic to a lexicographic sum over a fractal induced subgraph of Γ^ω.

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Benoit Larose

UQAM *Dismantlability, connectivity and finite duality*

We present analogs and extensions of results of Brightwell and Winkler on the relationship between dismantlability, connectivity in homomorphism graphs and finite duality of relational structures.

Work in progress, in collaboration with R. Briceño (Tel Aviv), A. Bulatov (Simon Fraser) and V. Dalmau (Pompeu Fabra).

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Martin Loebl

Prague

Precise complexity of rainbow even perfect matchings

I will introduce a variant o f the perfect matching problem and discuss its complexity and relation to max cut problem in embedded graphs.

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Gary MacGillivray

Victoria

Homomorphically full graphs

A graph G is *homomorphically full* if every homomorphic image of G is a subgraph of G. The homomorphically full simple graphs are characterized by six equivalent conditions, one of which is that they are the cographs with no induced $2K_2$. The homomorphically full reflexive graphs are also characterized by six equivalent conditions, one of which is that they are the threshold graphs. The situation is more complicated for digraphs. We show that conditions that are equivalent for undirected graphs are no longer equivalent.

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Serguei Norine

Montreal

Clustered coloring of minor-closed graph classes

Abstract: The *clustered chromatic number* of a graph class is the minimum integer t such that for some C the vertices of every graph in the class can be colored in t colors so that every monochromatic component has size at most C. We will discuss the clustered chromatic number of the class of graphs embeddable on a fixed surface and of general minor-closed classes. We will also mention the list version of the concept and connections of our results to bootstrap percolation.

Based on joint work with Zdeněk Dvořák.

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Arnaud Pecher

Bordeaux

Loász's theta function and perfect graphs

The clique number is a trivial lower bound for the chromatic number, but by far not always tight. The graphs where both parameters coincide for all induced subgraphs are called *perfect*. A main result of combinatorial optimization is that clique and chromatic number of a perfect graph are computable in polynomial time (Grötschel, Lovász and Schrijver 1981). In this short talk, I will present the proof of this celebrated result, with a focus on Lovász's theta function.

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Maurice Pouzet

Lyon

Free monoids and generalized metric spaces

Let A be an ordered alphabet, A^* be the free monoid over A ordered by the Higman ordering, and let $F(A^*)$ be the set of final segments of A^*. With the operation of concatenation, this set is a monoid. We show that the submonoid $F^\circ(A^*) := F(A^*) \setminus \{\emptyset\}$ is free. The MacNeille completion $N(A^*)$ of A^* is a submonoid of $F(A^*)$. As a corollary, we obtain that the monoid $N^\circ(A^*) := N(A^*) \setminus \{\emptyset\}$ is free. We give an interpretation of the freeness of $F^\circ(A^*)$ in the category of metric spaces over the Heyting algebra $V := F(A^*)$, with the non-expansive mappings as morphisms. Each final segment F of A^* yields the injective envelope $\mathcal{S}_F$ of a two-element metric space over V. The uniqueness of the decomposition of F is due to fact that the block decomposition of the graph $\mathcal{G}_F$ associated to this injective envelope is a path.

References

[1] L.M. Blumenthal, *Theory and applications of distance geometry,* Second edition Chelsea Publishing Co., New York 1970 xi+347 pp.

[2] L.M. Blumenthal, K. Menger, *Studies in geometry,* W. H. Freeman and Co., San Francisco, Calif. 1970 xiv+512 pp.

[3] A. Hudry, Injective envelope and parallel decomposition of a transition system, Discrete Math. 289 (2004), no. 1-3, 45–61.

[4] E. Jawhari, D. Misane, M. Pouzet, *Retracts graphs and ordered sets from the metric point of view,* (I.Rival ed) Contemporary Mathematics, Vol 57,175-226. 1986.

[5] M. Kabil, M. Pouzet, *Injective envelope of graphs and transition systems,* Discrete Math.192 (1998), 145-186.

[6] M. Pouzet, I.G. Rosenberg, *General metrics and contracting operations,* Graphs and combinatorics (Lyon, 1987; Montreal, PQ, 1988). Discrete Math. 130 (1994),103–169.

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Andre Raspaud

Bordeaux

List star edge coloring of graphs

A *star edge coloring* of a graph G is a proper edge coloring such that every 2-colored connected subgraph of G is a path of length at most 3. For a graph G, let the *list star chromatic index* of G, $ch'_0(G)$, be the minimum k such that for any k-uniform list assignment L for the set of edges, G has a star edge coloring from L. In this talk we will give a short survey of the different graph colorings close to the star edge coloring. We will present new results we obtained for the list star chromatic index of subcubic graphs and also bounds of the list star chromatic index in terms of maximum degree for sparse graphs.

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Norbert Sauer

Calgary

Forbidden finite structures, Ramsey theory, groups and topological dynamics

A survey.

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Bruce Shepherd

Montreal

Conflict-free disjoint paths and stable matchings

We outline a subroutine needed in the recent work on confluent flows with Adrian Vetta and Gord Wilfong. It asks for flows with the following flavour. Nodes are partitioned into "conflict cliques" and we may use at most one node from any given clique.

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Claude Tardif

Kingston

Colouring graphs with no odd cycles

A graph with no odd cycles can be coloured with two colours. That's all. Unless of course if I start talking about infinite graphs. And as a matter of fact, that's precisely what I intend to do. The purpose of the talk is to connect the very modern topic of constraint satisfaction problems with very ancient combinatorial axioms in set theory. This is joint work with Danny Rorabaugh and David Wehlau.

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Jan Volec

Montreal

On degree thresholds of cycles in oriented graphs

Motivated by Caccetta-Haggkvist conjecture, Kelly, Kuhn and Osthus initiated the study of minimum out-degree and semi-degree conditions that force an oriented graph to contain an oriented cycle of a given length. In particular, they proved for every l>=4 that if G is a sufficiently large n-vertex oriented graph with semi-degree > n/3, then G contains an oriented cycle of length l. It is easy to show that the bound is sharp for every l not divisible by 3. However, they conjectured that for l>=4 which is a multiple of 3, one can always do better. The smallest open case, which has drawn quite some attention and was a few times mentioned in open problem sessions by Kuhn and Osthus, is the one when l=6.

In this talk, we will prove for every l>=6 which is a multiple of 3 that if G is a sufficiently large n-vertex oriented graph with semi-degree > n/4, then G contains an oriented cycle of length l. The bound n/4 is again sharp, since blow-ups of oriented 4-cycle contain no cycle of length divisible by 3.

This is joint work with Roman Glebov and Andrzej Grzesik.

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Doug West

Chicago and Jinhua

Coloring, sparseness, and girth

We construct existence and sharpness examples for several questions in coloring and list coloring, using sparse graphs constructed from very tall trees. An *r-augmented tree* consists of a rooted tree plus edges added from each leaf to r ancestors. For $d, g, r \in \mathbb{N}$, we inductively construct a bipartite r-augmented complete d-ary tree having girth at least g. The height of such trees must grow extremely rapidly in terms of the girth.

Applications of this construction involve expanding the leaves of an r-augmented tree into special graphs. We use it for the following: (1) A new simple construction of graphs (and uniform hypergraphs) with large girth and chromatic number (not using hypergraphs with large edges). (2) Construction of large-girth bipartite graphs that are not k-choosable even though all proper subgraphs have average degree at most $2(k-1)$ (maximum average degree at most $2(k-1)$ makes a bipartite graph k-choosable). (3) Construction of a bipartite graph with large girth having a k-uniform list assignment L from which no proper coloring can be chosen even though the lists at adjacent vertices have only one common element (having two common elements guarantees L-colorability). (4) Enhancement of (2) so that the union of the lists has size $2k-1$ (size at most $2k-2$ guarantees L-colorability).

These results are joint work with Noga Alon, Alexandr V. Kostochka, Benjamin Reiniger, and Xuding Zhu.

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Peter Winkler

Dartmouth

Some pretty arguments using discrete probability

The beauty and power of discrete probability can be used to solve problems—including some that don't seem to have an probability in them—in elegant fashion, often without any calculation. We'll present several such problems, and answer most of them with pure reasoning.

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Hehui Wu
Shanghai

Digraphs coloring and tournaments with large domination number

A proper coloring of a digraph is a vertex coloring with no monochromatic directed cycle. Erdős and Neumann-Lara conjectured that if a undirected graph has large chromatic number, so does one of its orientaion. With Bojan Mohar, we proved the fractional version.

Recently, with Ararat Harutyunyan, Tien-Nam Le, Stéphan Thomassé, we show that if the domination number of a Tournament is large enough, then there exists a subtournament with big chromatic number and bounded size. This verifies a conjecture of Berger et al: If the out-neighborhood of each vertex of a tournament has bounded chromatic number, so does the whole tournament.