Polynomial Correlations: 
New Tools for Dependency Calibration

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The correlation of a bivariate random variable is the expected product of two polynomials. These polynomials are the first of two respective polynomial sequences, each of which is an orthonormal sequence with respect to the marginal distributions. This paper shows how expected products of higher order orthonormal polynomials can capture further aspects of dependence. These include convexity, the tendency for high values of one variable to be associated with extreme (high or low) values of another, and arachnitude which is the tendency of extreme (high or low) values of two variables to occur simultaneously.

We also develop rank versions of these quantities. Alongside the familiar rank correlation, we have rank convexity and rank arachnitude. Sample versions of these statistics can be useful in the calibration of copulas.

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