# Procyclicality of Empirical Measurements of Risk in Financial Markets


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# Motivation

- The capital computation is usually done by financial institutions once a year and looked at it in a **static way**, based on **past data**
- How well the risk assessment (capital) holds in the future?
- Accepted idea: **risk measurements** made with 'regulatory' risk measures, are **pro-cyclical**
  - $\nearrow$ in times of crisis, they overestimate the future risk
  - $\searrow$ they underestimate it in quiet times

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# Three aims

Introducing a **new approach** of studying risk measurements, we

1. generalize in a simple way the static 'regulatory' risk measure VaR to a **dynamic one**
2. test not only the relevance of the SQP risk measure, but also its **predictive power**
3. **quantify empirically procyclicality**

For this:

1. Consider the measurement itself as a stochastic process, introducing **Sample Quantile Process** (SQP) as a risk measure to ...

   ... compare it to the VaR

   ... test its use in practice

2. (a) Play with the random measure defining the SQP

   (b) Define a **look-forward ratio** to ...

   ... see if the historical estimate of the SQP overestimates the risk in times of high volatility

3. Use the **realized volatility** as a marker for the **market state**, ...

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# Risk Measures: VaR

- In financial markets, the most popular risk measure is **Value-at-Risk** (VaR)

- Given a loss random variable $L$ (with cdf $F_L$), level $\alpha \in (0,1)$

$$\mathrm{VaR}(\alpha) = \inf\{\mathrm{x} \in \mathbb{R} : \mathbb{P}(\mathrm{L} \leq \mathrm{x}) \geq \alpha\} \overset{\mathrm{F_L contin.}}{=} \mathrm{F}_{\mathrm{L}}^{-1}(\alpha)$$

- Practically, VaR is estimated as an **empirical quantile**:
Given a sample of $n$ **historical losses** $(L_1, \cdots, L_n)$, $\alpha \in (0,1)$, VaR is computed as:

$$F_{n;L}^{-1}(\alpha) = \inf \left\{ x : \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}_{(L_i \leq x)} \geq \alpha \right\}.$$

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# 1 - Sample Quantile Process

- The idea of a **Sample Quantile Process** (SQP) was first introduced by Miura (1992), then followed by Akahori (1995), Embrechts & Samorodnitsky (1995): Given a stochastic process $L = (L_t, t \geq 0)$, a level $\alpha \in (0,1)$, a fixed time frame $T$ and a random measure $\mu$ on $\mathbb{R}^+$, the SQP is defined at time $t$ as

$$Q_{T,\alpha,t}(L) = \inf\left\{x : \frac{1}{\int_{t-T}^{t} \mu(s)ds} \int_{t-T}^{t} \mathbb{1}_{(L_s \leq x)} \mu(s)ds \geq \alpha\right\}.$$

- For $\mu =$ **Lebesgue measure**, it corresponds to a **VaR process** $(Q_{T,\alpha,t}(L))_t$:

$$Q_{T,\alpha,t}(L) = \inf\left\{x : \frac{1}{T} \int_{t-T}^{t} \mathbb{1}_{(L_s \leq x)} ds \geq \alpha\right\}$$

- The corresponding empirical estimator is:

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# SQP Beyond VaR

The choice of measure $\mu$ is extended to account for other possibilities

- We consider e.g. $\mu(s) = |L_s|^p$, with $p$ a free parameter:

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- $p = 0$: the VaR process (with a rolling window)
  $p < 0$: SQP puts more weight to the center of the distribution
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- We consider e.g. $\mu(s) = |L_s|^p$, with $p$ a free parameter:

$$Q_{T,p,\alpha,t}(L) = \inf\left\{x : \frac{1}{\int_{t-T}^{t} |L_s|^p ds} \int_{t-T}^{t} \mathbb{1}_{(L_s \leq x)} |L_s|^p ds \geq \alpha\right\}.$$

- $p = 0$: the VaR process (with a rolling window)
  $p < 0$: SQP puts more weight to the center of the distribution
  $p > 0$: SQP puts more weight to the tail of the distribution

# Empirical Setup

- Data: **for 11 stock indices**, daily log-returns from 1987 to 2016
- Fix $T$= 1 or 3 years
  Evaluate the SQP on a monthly frequency via $\widehat{Q}_{T,p,1-\alpha,t}(X)$ (the results for 3 years are essentially the same as for 1 year)

  Aesthetic remark: We use log-returns $X = (X_i)_i$. Hence $Q_{T,p,\alpha,t}(L)$ (e.g. with $\alpha = 95\%$) corresponds to $Q_{T,p,1-\alpha,t}(X)$ (since $L = -X$).
- We vary the following parameters of the SQP:
  ...the parameter $p$ of the measure $\mu(s) = |L_s|^p$, $p = 0, 0.5, 1, 2$
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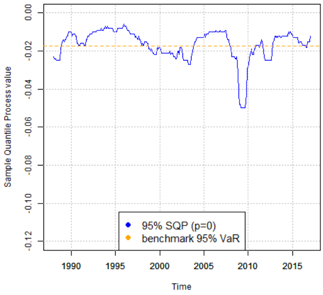
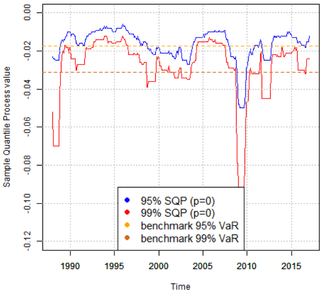
# Observations on the S&P 500 (T=1)

- Left: SQP for fixed $\alpha = 0.95$ and varying $p = 0, 0.5, 1, 2$
- Right: Comparison of $\alpha = 0.95, 0.99$ for $p = 0, 0.5, 1, 2$



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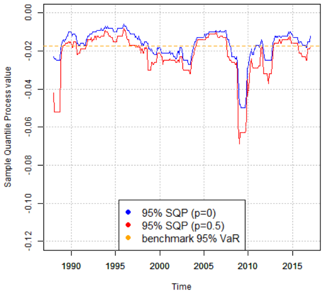


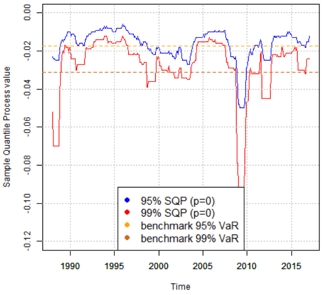

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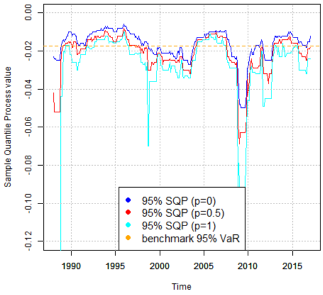


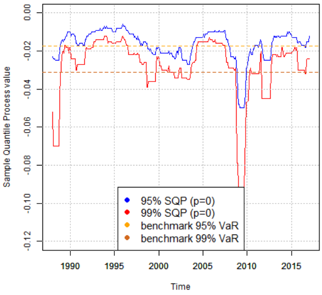

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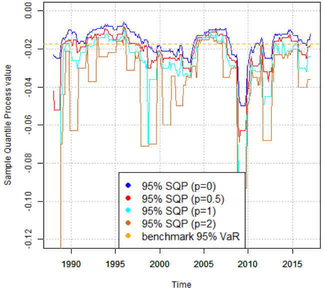


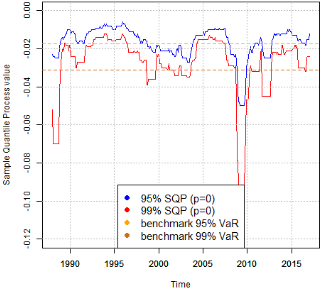

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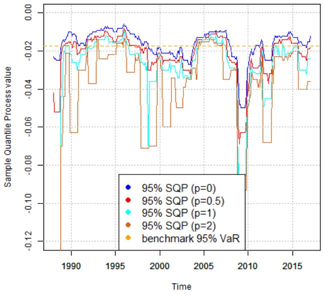


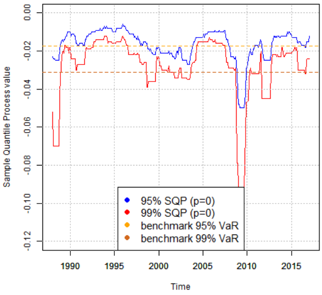

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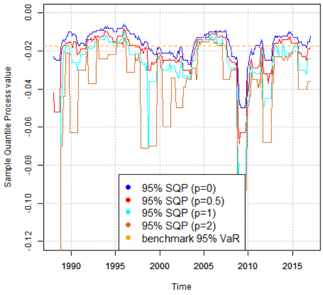


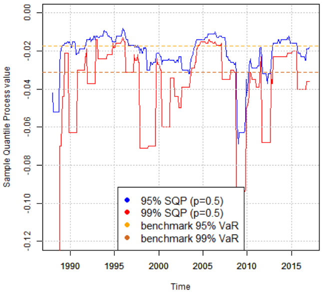

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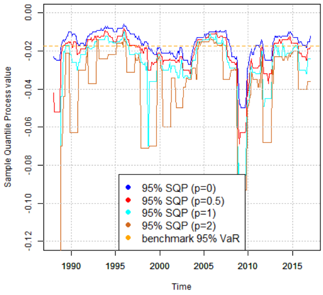


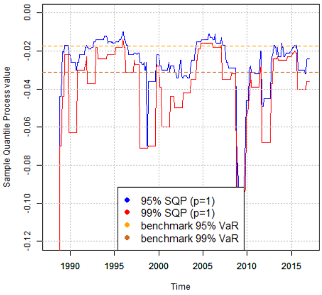

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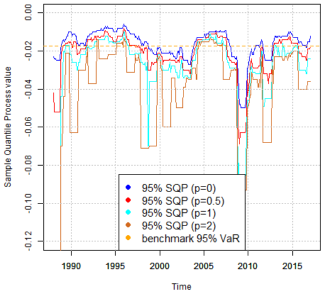


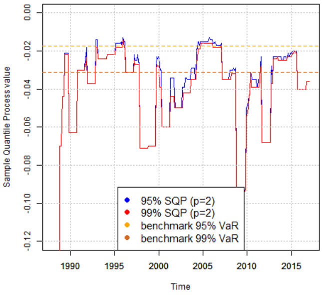

# Observations on the SQP Estimates

General observation over all 11 stock indices

- For fixed level $\alpha$: the higher the power p, the lower the SQP
- The higher the power p, the more volatile the SQP
- Difference between the SQP for $\alpha = 95\%$ and $\alpha = 99\%$ diminishes for increasing p
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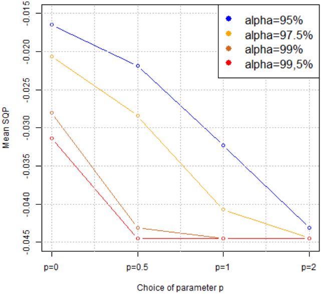
Mean SQP over the whole sample (1987-2016) for S&P500 (T=1)



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- Introduce a **new quantity**: *look-foward ratio* of SQP's

$$R_{T,p,\alpha,t} = \frac{\hat{Q}_{1,0,\alpha,\mathbf{t+1y}}}{\hat{Q}_{T,p,\alpha,t}}$$

... $\hat{Q}_{T,p,\alpha,t}$ is **used as a predictor** of the risk one year later $(t + 1y)$
... $\hat{Q}_{1,0,\alpha,\mathbf{t+1y}}$ is the estimated **realized** risk at time $t + 1$ **(a posteriori)** (empirical VaR on 1 year, as asked by regulators)

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# Observations on the Average Ratios

| Ratio | AUS | CAN | FRA | DEU | ITA | JPN | NLD | SGP | SWE | GBR | USA | AVG$\pm\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha = 0.95$ | | | | | | | | | | | | |
| $p = 0$ | 1.05 | 1.10 | 1.09 | 1.12 | 1.10 | 1.12 | 1.11 | 1.07 | 1.10 | 1.08 | 1.07 | **1.09 $\pm$ 0.02** |
| $p = 0.5$ | 1.04 | 1.12 | 1.09 | 1.12 | 1.10 | 1.16 | 1.11 | 1.07 | 1.10 | 1.08 | 1.06 | **1.10 $\pm$ 0.03** |
| $p = 1$ | 1.16 | 1.13 | 1.07 | 1.27 | 1.15 | 1.24 | 1.12 | 1.19 | 1.11 | 1.10 | 1.09 | **1.15 $\pm$ 0.06** |
| $p = 2$ | 1.26 | 1.14 | 1.11 | 1.26 | 1.15 | 1.30 | 1.11 | 1.19 | 1.14 | 1.12 | 1.18 | **1.18 $\pm$ 0.07** |
| $\alpha = 0.99$ | | | | | | | | | | | | |
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| $p = 0.5$ | 1.25 | 1.14 | 1.11 | 1.27 | 1.16 | 1.30 | 1.12 | 1.21 | 1.14 | 1.13 | 1.17 | **1.18 $\pm$ 0.07** |
| $p = 1$ | 1.22 | 1.12 | 1.10 | 1.22 | 1.13 | 1.23 | 1.09 | 1.15 | 1.11 | 1.10 | 1.14 | **1.15 $\pm$ 0.05** |
| $p = 2$ | 1.22 | 1.12 | 1.10 | 1.22 | 1.13 | 1.23 | 1.09 | 1.15 | 1.11 | 1.10 | 1.14 | **1.15 $\pm$ 0.05** |

- Average ratios increase with $\alpha$, indicating an increased **underestimation in the tails**
- Underestimation increases with $p$
- Average Ratios clearly above 1, i.e. do not predict future risk well
- The average does not tell the whole story (missing the dynamics)

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| $p = 0$ | 1.04 | 1.10 | 1.06 | 1.11 | 1.11 | 1.14 | 1.11 | 1.13 | 1.10 | 1.09 | 1.08 | **1.10 $\pm$ 0.03** |
| $p = 0.5$ | 1.25 | 1.14 | 1.11 | 1.27 | 1.16 | 1.30 | 1.12 | 1.21 | 1.14 | 1.13 | 1.17 | **1.18 $\pm$ 0.07** |
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| $\alpha = 0.95$ | | | | | | | | | | | | |
| $p = 0$ | 1.05 | 1.10 | 1.09 | 1.12 | 1.10 | 1.12 | 1.11 | 1.07 | 1.10 | 1.08 | 1.07 | **1.09 $\pm$ 0.02** |
| $p = 0.5$ | 1.04 | 1.12 | 1.09 | 1.12 | 1.10 | 1.16 | 1.11 | 1.07 | 1.10 | 1.08 | 1.06 | **1.10 $\pm$ 0.03** |
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# The Predictive Power of the SQP

- Consider the following Root Mean Square Error (RMSE) for different values of $p$ and $\alpha$:

$$RMSE := \sqrt{\frac{1}{N}\sum_{t=1}^{N}(R_{T,p,\alpha,t} - 1)^2},$$

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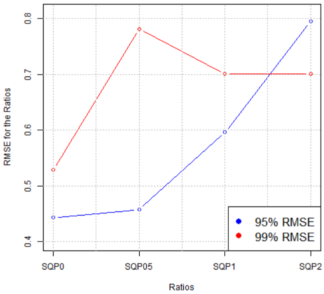

# Observations on the RMSE

Take e.g. $T = 1$ year

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha = 0.95$ | | | | | | | | | | | | |
| $p = 0$ | 0.38 | 0.48 | 0.46 | 0.52 | 0.50 | 0.57 | 0.53 | 0.48 | 0.49 | 0.45 | 0.44 | **0.48 $\pm$ 0.05** |
| $p = 0.5$ | 0.38 | 0.61 | 0.48 | 0.57 | 0.51 | 0.70 | 0.59 | 0.54 | 0.52 | 0.48 | 0.46 | **0.53 $\pm$ 0.09** |
| $p = 1$ | 0.81 | 0.67 | 0.45 | 1.31 | 0.73 | 0.88 | 0.62 | 0.93 | 0.60 | 0.52 | 0.60 | **0.74 $\pm$ 0.24** |
| $p = 2$ | 0.95 | 0.70 | 0.53 | 1.18 | 0.70 | 1.06 | 0.60 | 0.91 | 0.61 | 0.57 | 0.79 | **0.78 $\pm$ 0.22** |
| $\alpha = 0.99$ | | | | | | | | | | | | |
| $p = 0$ | 0.38 | 0.57 | 0.40 | 0.52 | 0.53 | 0.62 | 0.59 | 0.73 | 0.53 | 0.48 | 0.53 | **0.54 $\pm$ 0.10** |
| $p = 0.5$ | 0.94 | 0.71 | 0.55 | 1.18 | 0.71 | 1.06 | 0.63 | 0.94 | 0.62 | 0.60 | 0.78 | **0.79 $\pm$ 0.21** |
| $p = 1$ | 0.85 | 0.65 | 0.49 | 0.96 | 0.61 | 0.87 | 0.53 | 0.77 | 0.55 | 0.51 | 0.70 | **0.68 $\pm$ 0.16** |
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- The averages tell us that the future risk is not well assessed with our risk measures
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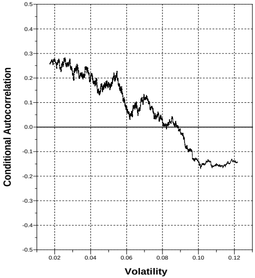
Inspiration (Dacorogna et al. (2001)):

- In times of **low volatility**, consecutive returns tend to be **positively auto-correlated**,
- in times of **high volatility**, consecutive returns tend to be **negatively auto-correlated**



# Realized Volatility

We defined a **realized volatility** indicator (**annualized**) as:

$$v_t = \hat{\sigma}(t) := \sqrt{\frac{1}{N-1} \sum_{i=t-N+1}^{t} \left( X_i - \frac{1}{N} \sum_{i=t-N+1}^{t} X_i \right)^2} * \sqrt{252}$$

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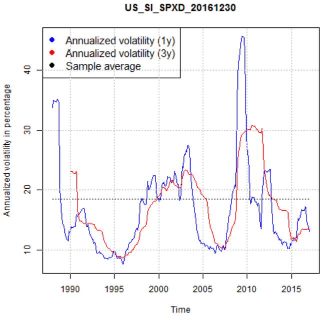
The realized volatility for $T = 1$ and $3$ years of the S&P 500 from 1987 to 2016 (annualized)



# Empirical Properties of the Realized Volatility

- We can see that the realized volatility is above the sample average (benchmark) for periods of high market instability or crisis (and not only for crises in the USA)
- The high volatility of the period 1987-1989 is for instance explained by the New York Stock Exchange crash in October 1987
- In 1997, Asia was hit by a crisis, as well as Russia in 1998 and Argentina in 1999-2000
- In 2001, the United States experienced the bursting of the internet bubble
- Following the Lehman Brother's bankruptcy, the period of 2008-2009 was a period of very high volatility
- Finally, the sovereign debt crisis in Europe also impacted the S&P 500 Index in 2011-2012. This is an illustration of the increased dependence between the markets during times of crisis

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# Empirical Properties of the Realized Volatility

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# Realized Volatility as a Proxy for Market States

- Realized volatility is a **reasonable proxy** to qualify the times of high risks and to discriminate between quiet and crisis periods

- We will use it to **condition our statistics** and see if we can detect different behaviors of the price process during these periods in comparison to quiet times. Conditioning the variable of interest on volatility was the basis of the study in (Dacorogna et al. (2001))

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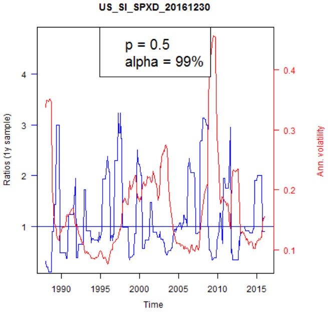
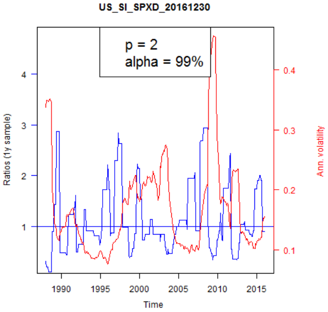
# Relation between Volatility and SQP Ratios

- Qualitative view:



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- The look-forward ratio is **above 1** if volatility is **low** (risk is underestimated, if past volatility is low)
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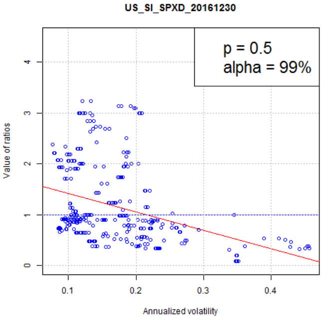

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**Linear correlation** $\rho(\log(R_t), v_t)$:

| | Pearson correlation, T=1y | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AUS | CAN | FRA | DEU | ITA | JPN | NLD | SGP | SWE | GBR | USA | AVG$\pm\sigma$ |
| $\alpha = 0.95$ | | | | | | | | | | | | |
| $p = 0$ | -0.52 | -0.51 | -0.51 | -0.49 | -0.48 | -0.64 | -0.55 | -0.51 | -0.58 | -0.54 | -0.55 | -0.54 $\pm$ 0.05 |
| $p = 0.5$ | -0.67 | -0.50 | -0.48 | -0.44 | -0.40 | -0.69 | -0.57 | -0.62 | -0.53 | -0.60 | -0.56 | -0.55 $\pm$ 0.09 |
| $p = 1$ | -0.67 | -0.49 | -0.41 | -0.24 | -0.24 | -0.50 | -0.56 | -0.65 | -0.41 | -0.59 | -0.60 | -0.49 $\pm$ 0.15 |
| $p = 2$ | -0.66 | -0.45 | -0.34 | -0.17 | -0.21 | -0.48 | -0.42 | -0.58 | -0.31 | -0.49 | -0.49 | -0.42 $\pm$ 0.15 |
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| Volatility year $t$ | SQP ratio | Meaning |
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# Looking for Explanations

- We look for the simplest model that could explain the empirical behavior of SQP with volatility

- GARCH models are very popular for modelling the clustering of volatility and its return to the mean, in financial returns

- We choose to use the **simplest version** GARCH(1,1) to isolate the effect of clustering

$$\sigma_t^2 = \omega + \alpha\, r_{t-1}^2 + \beta\, \sigma_{t-1}^2$$

  where $r_{t+1} = \epsilon_t\, \sigma_t$ and $\epsilon_t \in \mathcal{N}(0,1)$

- We fit the parameters $\omega, \alpha, \beta$ to each full sample of the 11 indices, using a **robust optimization method** (Zumbach 2000)

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# GARCH Parametrization

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|---|---|---|---|---|---|---|---|---|---|---|---|
| *GARCH Parameters* | | | | | | | | | | | |
| $\omega$ $[10^{-6}]$ | 4.13 | 1.32 | 3.11 | 2.51 | 2.66 | 5.96 | 2.63 | 3.42 | 4.08 | 2.10 | 2.24 |
| $\alpha$ $[10^{-1}]$ | 1.66 | 1.08 | 0.98 | 0.95 | 0.83 | 1.48 | 1.12 | 1.33 | 1.12 | 1.13 | 1.07 |
| $\beta$ $[10^{-1}]$ | 7.93 | 8.79 | 8.83 | 8.89 | 9.02 | 8.19 | 8.74 | 8.48 | 8.65 | 8.68 | 8.76 |
| $\alpha + \beta$ | *0.96* | *0.99* | *0.98* | *0.98* | *0.99* | *0.97* | *0.99* | *0.98* | *0.98* | *0.98* | *0.98* |
| *Fitting Results* | | | | | | | | | | | |
| Likelihood $[10^{4}]$ | 2.54 | 2.58 | 2.35 | 2.39 | 2.32 | 2.25 | 2.37 | 2.36 | 2.30 | 2.51 | 2.45 |
| Volatility [%] | 16.3 | 16.7 | 20.4 | 20.2 | 20.2 | 20.8 | 22.4 | 21.8 | 21.0 | 16.1 | 17.8 |
| Historical [%] | 16.0 | 15.9 | 20.0 | 20.1 | 21.1 | 21.3 | 21.6 | 21.0 | 21.1 | 16.7 | 18.5 |

- We see that the optimization gives in all the cases parameters that keep the GARCH stationary $(\alpha + \beta < 1)$
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# GARCH Parametrization

| | AUS | CAN | FRA | DEU | ITA | JPN | NLD | SGP | SWE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *GARCH Parameters* | | | | | | | | | | | |
| $\omega$ $[10^{-6}]$ | 4.13 | 1.32 | 3.11 | 2.51 | 2.66 | 5.96 | 2.63 | 3.42 | 4.08 | 2.10 | 2.24 |
| $\alpha$ $[10^{-1}]$ | 1.66 | 1.08 | 0.98 | 0.95 | 0.83 | 1.48 | 1.12 | 1.33 | 1.12 | 1.13 | 1.07 |
| $\beta$ $[10^{-1}]$ | 7.93 | 8.79 | 8.83 | 8.89 | 9.02 | 8.19 | 8.74 | 8.48 | 8.65 | 8.68 | 8.76 |
| $\alpha + \beta$ | *0.96* | *0.99* | *0.98* | *0.98* | *0.99* | *0.97* | *0.99* | *0.98* | *0.98* | *0.98* | *0.98* |
| *Fitting Results* | | | | | | | | | | | |
| Likelihood $[10^4]$ | 2.54 | 2.58 | 2.35 | 2.39 | 2.32 | 2.25 | 2.37 | 2.36 | 2.30 | 2.51 | 2.45 |
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# Using the GARCH Process to Test the Log SQP-Ratios

- **100 replications** for each index with **same sample size** as the data
- On each, apply the **same statistical analyses** as on historical data
- Qualitatively **similar results** as with historical data

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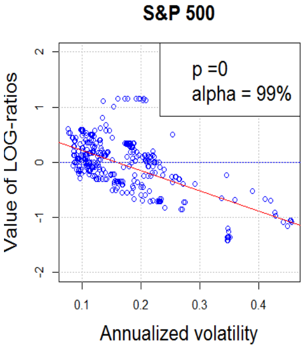

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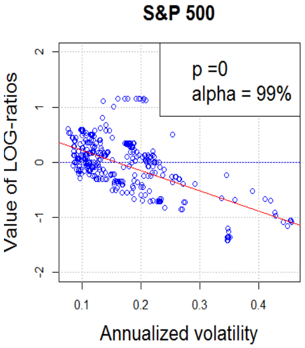


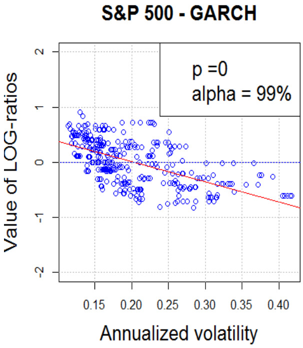

# Results for the GARCH

| | Pearson Correlation, T=1y, average of 100 GARCH simulations | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AUS | CAN | FRA | DEU | ITA | JPN | NLD | SGP | SWE | GBR | USA | AVG$\pm\sigma$ |
| $\alpha = 0.95$ | | | | | | | | | | | | |
| $p = 0$ | -0.63 | -0.62 | -0.63 | -0.62 | -0.61 | -0.64 | -0.63 | -0.64 | -0.64 | -0.63 | -0.63 | -0.63 $\pm$ 0.05 |
| $p = 0.5$ | -0.68 | -0.64 | -0.64 | -0.63 | -0.61 | -0.68 | -0.64 | -0.66 | -0.66 | -0.65 | -0.64 | -0.65 $\pm$ 0.07 |
| $p = 1$ | -0.68 | -0.63 | -0.62 | -0.61 | -0.60 | -0.68 | -0.62 | -0.66 | -0.65 | -0.64 | -0.63 | -0.64 $\pm$ 0.08 |
| $p = 2$ | -0.63 | -0.58 | -0.58 | -0.57 | -0.55 | -0.63 | -0.59 | -0.62 | -0.61 | -0.60 | -0.59 | -0.59 $\pm$ 0.09 |
| $\alpha = 0.99$ | | | | | | | | | | | | |
| $p = 0$ | -0.63 | -0.60 | -0.60 | -0.59 | -0.58 | -0.62 | -0.60 | -0.62 | -0.61 | -0.61 | -0.60 | -0.61 $\pm$ 0.06 |
| $p = 0.5$ | -0.60 | -0.57 | -0.57 | -0.56 | -0.56 | -0.60 | -0.58 | -0.60 | -0.58 | -0.58 | -0.58 | -0.58 $\pm$ 0.08 |
| $p = 1$ | -0.59 | -0.58 | -0.56 | -0.56 | -0.55 | -0.59 | -0.58 | -0.59 | -0.58 | -0.58 | -0.58 | -0.58 $\pm$ 0.08 |
| $p = 2$ | -0.59 | -0.58 | -0.56 | -0.56 | -0.55 | -0.59 | -0.58 | -0.59 | -0.58 | -0.58 | -0.58 | -0.58 $\pm$ 0.09 |

- Negative correlation **- as for historical data**
- negative correlations are higher $(0.58 - 0.63)$ than in data $(0.42 - 0.57)$
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| | AUS | CAN | FRA | DEU | ITA | JPN | NLD | SGP | SWE | GBR | USA | AVG$\pm\sigma$ |
| $\alpha = 0.95$ | | | | | | | | | | | | |
| $p = 0$ | -0.63 | -0.62 | -0.63 | -0.62 | -0.61 | -0.64 | -0.63 | -0.64 | -0.64 | -0.63 | -0.63 | -0.63 $\pm$ 0.05 |
| $p = 0.5$ | -0.68 | -0.64 | -0.64 | -0.63 | -0.61 | -0.68 | -0.64 | -0.66 | -0.66 | -0.65 | -0.64 | -0.65 $\pm$ 0.07 |
| $p = 1$ | -0.68 | -0.63 | -0.62 | -0.61 | -0.60 | -0.68 | -0.62 | -0.66 | -0.65 | -0.64 | -0.63 | -0.64 $\pm$ 0.08 |
| $p = 2$ | -0.63 | -0.58 | -0.58 | -0.57 | -0.55 | -0.63 | -0.59 | -0.62 | -0.61 | -0.60 | -0.59 | -0.59 $\pm$ 0.09 |
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| $p = 0$ | -0.63 | -0.60 | -0.60 | -0.59 | -0.58 | -0.62 | -0.60 | -0.62 | -0.61 | -0.61 | -0.60 | -0.61 $\pm$ 0.06 |
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| $p = 1$ | -0.59 | -0.58 | -0.56 | -0.56 | -0.55 | -0.59 | -0.58 | -0.59 | -0.58 | -0.58 | -0.58 | -0.58 $\pm$ 0.08 |
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| $p = 0$ | -0.63 | -0.62 | -0.63 | -0.62 | -0.61 | -0.64 | -0.63 | -0.64 | -0.64 | -0.63 | -0.63 | -0.63 $\pm$ 0.05 |
| $p = 0.5$ | -0.68 | -0.64 | -0.64 | -0.63 | -0.61 | -0.68 | -0.64 | -0.66 | -0.66 | -0.65 | -0.64 | -0.65 $\pm$ 0.07 |
| $p = 1$ | -0.68 | -0.63 | -0.62 | -0.61 | -0.60 | -0.68 | -0.62 | -0.66 | -0.65 | -0.64 | -0.63 | -0.64 $\pm$ 0.08 |
| $p = 2$ | -0.63 | -0.58 | -0.58 | -0.57 | -0.55 | -0.63 | -0.59 | -0.62 | -0.61 | -0.60 | -0.59 | -0.59 $\pm$ 0.09 |
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| $p = 1$ | -0.68 | -0.63 | -0.62 | -0.61 | -0.60 | -0.68 | -0.62 | -0.66 | -0.65 | -0.64 | -0.63 | -0.64 $\pm$ 0.08 |
| $p = 2$ | -0.63 | -0.58 | -0.58 | -0.57 | -0.55 | -0.63 | -0.59 | -0.62 | -0.61 | -0.60 | -0.59 | -0.59 $\pm$ 0.09 |
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| $p = 0$ | -0.63 | -0.60 | -0.60 | -0.59 | -0.58 | -0.62 | -0.60 | -0.62 | -0.61 | -0.61 | -0.60 | -0.61 $\pm$ 0.06 |
| $p = 0.5$ | -0.60 | -0.57 | -0.57 | -0.56 | -0.56 | -0.60 | -0.58 | -0.60 | -0.58 | -0.58 | -0.58 | -0.58 $\pm$ 0.08 |
| $p = 1$ | -0.59 | -0.58 | -0.56 | -0.56 | -0.55 | -0.59 | -0.58 | -0.59 | -0.58 | -0.58 | -0.58 | -0.58 $\pm$ 0.08 |
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| $p = 1$ | -0.59 | -0.58 | -0.56 | -0.56 | -0.55 | -0.59 | -0.58 | -0.59 | -0.58 | -0.58 | -0.58 | -0.58 $\pm$ 0.08 |
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# Summary of the Results

1. We introduced a 'dynamic generalization of VaR' - the SQP using $\mu(s) = |L_s|^p$ for various values of $p$
   In this case low powers of $p$ are favorable to highlight the difference between the thresholds $\alpha$
   Also, the predictive power is better (although not good) for lower $p$

2. Pro-cyclicality of the SQP confirmed and quantified (by conditioning to realized volatility):
   During **high-volatility** periods, those risk measures **overestimate** the risks for the following years, whereas during **low-volatility** periods, they **underestimate** them

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# Conclusion

- Risk management imposed by regulators **favors pro-cyclical behaviors** of the market actors. Following a pure historical estimation of risk measures financial institutions in general would be **left unprepared** in case of crisis

- Consequently one should **enhance the capital** requirements **in quiet times** and **relax** them **during the crises**, i.e. we need to introduce anti-cyclical risk management measures

- These measures should be **in line with the empirical results** without hampering the system or introducing rules that weaken economic valuation of liabilities

- This is what we are currently developing: A design of the SQP (the random measure $\mu$) with the **proper dynamical behavior** as a good basis for anti-cyclical regulation

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