

1. An invitation to stochastic topology
2. A brief survey of random finitely-presented groups
3. The bouquet-of-spheres conjecture

Matthew Kahle^{*}

mkahle@math.osu.edu

1. There has been a lot of interest over the past decade in various kinds of stochastic topology: Random manifolds, random knots, etc. We will briefly discuss these models, and then focus our attention on random simplicial complexes, particularly on the Linial–Meshulam random 2-complex which is a 2-dimensional analogue of the Erdős–Rényi random graph. We will overview a number of phase transitions that are known to occur for this model.

2. We will introduce a few of the most studied models of random groups: The Gromov density model, the triangular model, and the random fundamental group. We will discuss their most fundamental properties, such as hyperbolicity, Kazhdan Property (T), and cohomological dimension. We are especially interested in comparing and contrasting various properties of the models, and in pointing out many open problems.

3. The clique complex of the Erdős–Rényi random graph is a simple model that puts a measure on a wide range of topologies. We will discuss the recent proof by Fowler, Hoffman, K., and Malen, that for wide range of parameter, such complexes are homotopy equivalent to bouquets of spheres with high probability. Indeed, for every $k \neq 2$, there is a regime where one expects the homotopy type of a bouquet of k -spheres w.h.p. This improves on several earlier results, but the proof is almost completely self contained. This provides a measure-theoretic explanation for the ubiquity of bouquets of spheres in topological combinatorics.

^{*}Department of Mathematics, The Ohio State University, 231 W. 18th Ave., Columbus, OH 43210, USA