Atelier
« Atelier : Méthodes probabilistes en topologie »
14-18 novembre 2016

Workshop
"Workshop: Probabilistic methods in topology"
November 14—18, 2016

Computer driven questions, pre-theorems and theorems in geometry

Moira Chas*
moira.chas@stonybrook.edu

Several numbers can be associated to a non-based or free homotopy class X of closed curves on a surface S with boundary and negative Euler characteristic. Among these,

- the self-intersection number of X (this is the smallest number of times a representative of the X crosses itself),
- the word length of X (given a minimal set of generators of the fundamental group, this is the smallest number of generators in a word representing the deformation or conjugacy class) and
- the length of the geodesic in X (given a hyperbolic metric on S with geodesic boundary)
- the number of free homotopy classes of a given word length the mapping class group orbit of X.

The interrelations of these numbers exhibit many patterns when explicitly determined or approximated by running a variety of algorithms in a computer.

We will discuss how these computations lead to counterexamples to existing conjectures and to the discovery of new patterns. Some of these new patterns, so intricate and unlikely that they are certainly true (even if not proven yet), are "pre-theorems". Many of these pre-theorems later became theorems. An example of such a theorem states that the distribution of the self-intersection of free homotopy classes of closed curves on a surface, appropriately normalized, sampling among given word length, approaches a Gaussian when the word length goes to infinity. An example of a counterexample (no pun untended!) is that there exists pairs of length equivalent free homotopy classes of curves on a surface S that have different self-intersection number. (Two free homotopy classes X and Y are length equivalent if for every hyperbolic metric M on S, $M(X) = M(Y)$).

*Department of Mathematics, Stony Brook University, Stony Brook, NY 11794-3651, USA