

Weighted Hurwitz numbers, τ -functions and topological recursion

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We review the notion of weighted single and double Hurwitz numbers, defined either combinatorially, as weighted enumerations of paths between conjugacy classes in the Cayley graph of the symmetric group S_n , or weighted enumerations of n -fold branched coverings of the Riemann sphere, with arbitrary classical or quantum weight generating functions $G(z)$. The corresponding generating functions for weighted Hurwitz numbers are shown to be KP or 2D-Toda τ -functions of hypergeometric type. For the case of semiclassical weight generating functions, for which the $G' = RG$, where R is rational, we derive differential equations with respect to the genus expansion parameter and the KP or Toda flow parameters that generalize the cut and join equations. An alternative way of generating the weighted Hurwitz numbers is the genus expansions of multi-current correlators, which are shown to satisfy the topological recursion relations corresponding to an associated spectral curve.

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