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Categorification and the Ising model

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In this talk I will explain the "Landau-Ginzburg approach to categorification" and, if time permits, its relation to fivebrane systems used in 3d-3d correspondence and related problems.

Landau-Ginzburg models in two dimensions are known for their simplicity on the one hand, and for their incredible effectiveness in a wide variety of problems, ranging from construction of exactly solvable string vacua to the study of critical phenomena in statistical physics. Their low "cost-benefit ratio" makes LG models very useful tools for building and exploring concrete examples of 2d conformal field theory (CFT).

The canonical example of a two-dimensional CFT is a supersymmetric A_{N-1} minimal models with central charge $c = 3 - \frac{6}{N+1}$. Such models are believed to have a Landau-Ginzburg description based on a single $\mathbb{C}^{N} = 2$ chiral superfield with a simple superpotential $W = x^{N+1}$. A similar superpotential that consists of k such terms, $W = x_1^{N+1} + x_2^{N+1} + \cdots + x_k^{N+1}$, reexpressed via the elementary symmetric functions of x_i is believed to describe Kazama-Suzuki coset model at level one with central charge $c = \frac{3k(N-k)}{N+1}$. These theories admit interesting half-BPS interfaces (defect lines / domain walls), whose junctions and webs (a.k.a. networks) we shall study.

Surprisingly, the fusion rules for interfaces in these LG models are precisely the relations on the web diagrams of Khovanov and Lauda in their work on categorification of quantum groups. The explanation of this surprising fact comes from realizing our LG model in a higher-dimensional system, either on the world-sheet of a surface operator in 4d theory or in 6d fivebrane theory. These higher-dimensional systems have already appeared in the physical construction of knot homologies, which makes the relation to Khovanov-Lauda webs and categorification of quantum groups not only natural but also unavoidable. In this framework, the LG model lives on "time × knot".

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The previous study of such higher-dimensional systems led to many concrete predictions, including new LG models for various colored knot homologies, some of which later were made into rigorous mathematical theories. For example, the above superpotential was proposed as a basic ingredient for categorifying knot invariants colored by the k-th anti-symmetric representation of sl(N). It plays a central role in categorification of skew Howe duality and in our story as well.

In the end I will list open questions and future prospects, including generalization to junctions of interfaces and surface operators colored by symmetric representations.

This talk is based on the ancient work with Johannes Walcher: `http://arxiv.org/pdf/hep-th/0512298.pdf` *as well as very recent work with Sungbong Chun and Daniel Roggenkamp*: `http://arxiv.org/pdf/1507.06318.pdf`.