

Kodaira fibrations and rigid surfaces

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Kodaira fibred surfaces are a remarkable example of projective classifying spaces, and there are still many intriguing open questions concerning them, especially the slope question, raised by Le Brun. I shall first describe some construction of double Kodaira fibres surfaces, done jointly with Rollenske, leading to the highest known slope, and to rigid Kodaira fibrations.

I shall then consider rigid surfaces, showing that they are either Del Pezzo surfaces of degree at least 5, or they are surfaces of general type. I shall then give a list of the known examples, which are all projective classifying spaces, and show some new examples obtained in collaboration with Ingrid Bauer. These include the BCD surfaces which recently gave counterexamples to an old question by Fujita concerning Variation of Hodge Structures (*joint work with Michael Dettweiler*).

After giving a criterion for a fibred surface to be a classifying space, I shall discuss some open question on fibred surfaces, for instance the existence of metrics of negative curvature on the BCD surfaces.

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